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Modified Leslie-Gower Model with Holling Type I Functional Responses and Cannibalism in Prey

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ABSTRACT

The predator-prey model is the mathematical model that describes the interaction behavior between prey and predator. This research discusses the modified Leslie-Gower model by considering the cannibalism behaviors of the prey that contains Holling type I response function, which is a predator with passive characteristics. The stability analysis stage includes determining the system's solution in the form of an equilibrium point, analyzing the local stability of each equilibrium using eigenvalues, and numerical simulation to synchronize the analysis results. Numerical simulations visualized in phase portraits with Python software. The results of the local stability analysis of the system obtained four equilibrium points, namely, equilibrium points E_1 dan E_3 are unstable while E_2 and E_4 is asymptotically stable with certain conditions. The results of numerical simulations show that only the equilibrium point E_4 which is asymptotically stable when the environment carries capacity parameters (e=2.1). Meanwhile, when e=2.878 then, only E_2 is asymptotically stable. In this research also using two different initial values, it is concluded that whatever the initial value used, the system solution always converges to the equilibrium points E_2 dan E_4. Changes in environmental carrying capacity affect the dynamics of system solutions.

Keywords: *Predator-prey model, Holling type I, Cannibalism*

Introduction

Predator-prey interaction in ecological systems is one of the interesting topics in ecological mathematics. The interaction between predator and prey can be modelled mathematically based on several factors, such as the growth rate of predator and prey, predator predation rate, and competition among predators. Competition is when two species use the same resources and compete for resources to survive [1].

Competition occurs among predators, and predation occurs among predators; the predators are cannibals. Cannibalism can occur due to size differences within a group and among similar predators [2]. According to [3], hunger can increase the tendency to cannibalize, and many animals commit Cannibalism only because of reduced food availability. Many natural species are cannibals [4]–[8], including fish, crabs, crayfish, spiders, polar bears, etc. Several researchers have discussed cannibalism models, including Basher et al. [9], who proposed the idea of the Holling Tanner predator-prey model with Holling type II response function and the cannibalism model on prey. Deng et al. [10] studied a Lotka Volterra predator-prey model incorporating predator cannibalism. Rayungsari et al. [11] also analyzed the predator-prey model, including predator cannibalism and refuge.

The response function is one of the most important components of the predator-prey relationship. Holling introduced a response function to the predator-prey model known as the Holling response function, which is the predator's predation rate on prey [12]. The response function can be divided into three types, namely Holling type I, type II, dan type III response functions. Type I response function occurs in predators that have passive characteristics. Type II response function occurs in predators characterized as actively searching for prey. Type III response function occurs when the prey population decreases, so the predator looks for another prey population. In this research, considering the Holling type I response function, the predator is a spider. Spiders are often generalized as highly cannibalistic animals [6]. One example of an interaction that illustrates this research is the spiders as predators and ladybugs as prey. Ladybugs are animals that have cannibalistic behavior. If they cannot get their normal food, they prey on their weak species, such as larvae or eggs [13].

Based on previous studies of predator-prey models, researchers are interested in reconstructing the models studied by considering the Leslie Gower model, in which predator populations grow logistically because the availability of prey populations limits them. This research considers the same behavior as previous researchers, namely the existence of Cannibalism. Unlike the model discussed by Deng et al, in this article, the growth rate of predator populations develops the Leslie Gower model by considering the environment's carrying capacity, called modified Leslie Gower [14], [15]. Ashine [16] compared two models modified by Leslie Gower, which consider prey refuge and those without prey refuge, using Holling type II. Prey population growth rate is affected by

Cannibalism and predation using a type I Holling response function. Dynamic analysis includes determining the solution of the system in the form of an equilibrium point, analyzing the local stability of each equilibrium using eigenvalues, and numerical simulation to synchronize the analysis results.

Research Methods

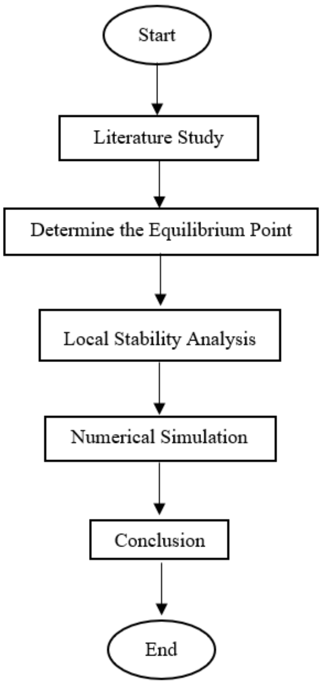

Figure 1. Research Process

Based on Figure 1, the first stage is a literature study by collecting sources from journals and articles related to previous research. The second stage is constructing a predator-prey model, extending the previous research study model. Furthermore, the third stage is dynamic analysis. Dynamic analysis includes determining the solution of the system in the form of an equilibrium point, the existence condition of the equilibrium point, analyzing the local stability of each equilibrium using eigenvalues, and numerical simulation to synchronize the analysis results using Python software.

Results dan Discussion

To construct the mathematical model, we set some assumptions:

1. Prey population growth rate is affected by Cannibalism and predation using a type I Holling response function.
2. The growth rate of prey decreases due to interactions by predators with predation rate $(\boldsymbol{a})$.
3. The growth rate of predator populations develops the Leslie Gower model by considering the carrying capacity of the environment $(\boldsymbol{e})$, which is called modified Leslie Gower.
4. The growth rate of predators increases due to interactions of prey with parameter $\boldsymbol{m}$ representing prey to the predator conversion factor.
5. Cannibalistic behavior only occurs in prey, not in predators.

The model was modified into:

$$\frac{dx}{dt} = x(b - ay - x + c_1 - cx),$$
$$\frac{dy}{dt} = y\left(r - \frac{my}{x+e}\right). \quad (1)$$

In this system (1), $x(t)$ is the prey population density dan y(t) is the predator population density. Parameters b, r, c_1, c are prey growth rate, predator growth rate, prey birth rate due to cannibalism behavior, and Cannibalism, respectively. All parameters in this system are positive, with $c_1 < c$.

Equilibrium Point

By the definition of equilibria, the equilibria of a system (1) are satisfied when.

$$\frac{dx}{dt} = \frac{dy}{dt} = 0$$

or

$$x(b - ay - x + c_1 - cx) = 0,$$

$$y\left(r - \frac{my}{x+e}\right) = 0. \quad (2)$$

based on equation (2), the following equilibrium points are obtained:

1. The equilibrium point of predator and prey extinction is $\mathrm{E}_1 = (0,0)$.
2. The equilibrium point of prey extinction is $\mathrm{E}_2 = \left(0, \frac{\mathrm{re}}{\mathrm{m}}\right)$.
3. The equilibrium point of predator extinction is $E_3 = \left(\frac{b+c_1}{c+1}, 0\right)$.
4. The equilibrium point when both predator and prey populations exist is. $E_4 = \left(-\frac{aer-bm-c_1m}{ar+cm+m}, \frac{r(ce+b+c_1+e)}{ar+cm+m}\right)$, conditionally E_4 exist if $e < \frac{m(b+c_1)}{ar}$.

Local Stability Analysis

The general form of the Jacobian matrix of the system (1) is given by:

$$J(x,y) = \begin{bmatrix} -ay - cx + b + c_1 - x + x(-c-1) & -xa \\ \frac{y^2m}{(x+e)^2} & r - \frac{2my}{x+e} \end{bmatrix}. \quad (3)$$

The stability of the equilibrium points of the system (1) are determined by the eigenvalues of the Jacobian matrix [17], and the result of (3) is

1. The equilibrium point $E_1 = (0,0)$ is always unstable with the type of stability is node if $\lambda_{1,}\lambda_2 > 0$, since the eigenvalues corresponding to this equilibrium point are $\lambda_1 = b + c_1 > 0$ and $\lambda_2 = r > 0$.
2. The equilibrium point $E_2 = \left(0, \frac{re}{m}\right)$ is asymptotically stable with the type of stability is node if $\lambda_{1,}\lambda_2 < 0$, since the eigenvalues corresponding to this equilibrium point are $\lambda_1 = -\frac{aer-bm-c_1m}{m} < 0$ and $\lambda_2 = -r < 0$, and to fulfil the stability condition, then $e > \frac{m(b+c_1)}{ar}$.
3. The equilibrium point $E_3 = \left(\frac{b+c_1}{c+1}, 0\right)$ is unstable with the type of stability saddle point if $\lambda_1 < 0 \wedge \lambda_2 > 0$, since the eigenvalues corresponding to this equilibrium point are $\lambda_1 = -b - c_1$ and $\lambda_2 = r$.
4. The equilibrium point $E_4 = \left(-\frac{aer-bm-c_1m}{ar+cm+m}, \frac{r(ce+b+c_1+e)}{ar+cm+m}\right)$ is asymptotically stable if $aer(c+1) < ar^2 + bcm + cc_1m + cmr + bm + c_1m + mr$ and $m(abr + ac_1r + bcm + bm + cc_1m + c_1m) > er(a^2r + acm + amr)$, since the eigenvalues corresponding to this equilibrium point are $\lambda_{1,2} = \frac{trace(A) \pm \sqrt{D}}{2}$, with $D = \left(trace(A)\right)^2 - 4det(A)$
   and
   $trace(A) = acemr + aemr - amr^2 - bcm^2 - cc_1m^2 - cm^2r - bm^2 - c_1m^2 - m^2r,$
   $det(A) = -a^2er^3 - acemr^2 + abmr^2 + ac_1mr^2 - aemr^2 + bcm^2r + bm^2r + cc_1m^2r + c_1m^2r.$

The stability of each equilibrium point can be seen in Table 1.

Table 1. Stability And Stability Conditions of The Equilibrium Point

Equilibrium Point	Stability	Stability Conditions
E_1	Unstable (*node*)	-

E_2	Asymptotically stable (*node*)	$e > \dfrac{m(b+c_1)}{ar}$
E_3	Unstable (*saddle point*)	-
E_4	Asymptotically stable	$trace(A) < 0$ and $det(A) > 0$

Numerical Simulation

This section describes the simulation of a system (1) based on parameter values. The selection of parameter values is based on the references and assumptions in Table 2 below.

Table 2. Parameter Values

Parameter	Description	Value	Reference
b	*Prey* growth rate	7	Assumption
c	Cannibalism	0.6	Assumption
c_1	Prey birth rate due to cannibalism behavior	0.4	Deng et al., 2019
a	Predation rate	0.6	Deng et al., 2019
r	Predator growth rate	1.2	Basher et al., 2016
m	Prey to the predator conversion factor	0.28	Assumption

In this article, the environment capacity parameter ($\boldsymbol{e}$) is varied to determine changes in the stability of several equilibrium points. Numerical simulations are performed by setting different values of the parameter e ($e = 2.1,\ e = 2.878$). In this simulation, two different initial value conditions are also given.

1. The first simulation with environmental carrying capacity parameter $e = 2.1$

   Based on the parameter values used in Table 2 with $e = 2.1$ four equilibrium points exist, namely $E_1 = (0,0), E_2 = (0,9), E_3 = (4.6,0), E_4 = (0.48,11.05)$. Based on the conditions of existence of the equilibrium point E_4 which is fulfilled when $e < \frac{m(b+c_1)}{ar}$ obtained $2.1 < 2.877$. Therefore, it is clear that E_4 exists. Analysis of stability with these parameter values, the eigenvalues of each equilibrium point are obtained as follows:

   a. $E_1 = (0,0)$, with $\lambda_1 = 1.2 > 0 \wedge \lambda_2 = 7.4 > 0$, equilibrium point E_1 is unstable.

   b. $E_2 = (0,9)$, with $\lambda_1 = -1.2 < 0 \wedge \lambda_2 = 2 > 0$, equilibrium point E_2 is unstable.

   c. $E_3 = (4.6,0)$, with $\lambda_1 = -7.4 < 0 \wedge \lambda_2 = 1.2 > 0$, titik kesetimbangan E_3 is unstable.

   d. $E_4 = (0.48,11.05)$ with $\lambda_1 = -0.98 + 1.19 < 0 \wedge \lambda_2 = -0.98 - 1.19 < 0$, equilibrium point E_4 is asymptotically stable.

   Thus, it is only the equilibrium point. E_4 which is asymptotically stable. See Figure 2 and Figure 3 below.

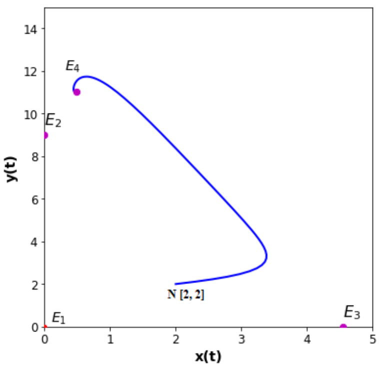

Figure 1. Phase Portraits of The System (1) With e=2.1 And Initial Value [2, 2] Goes To E_4.

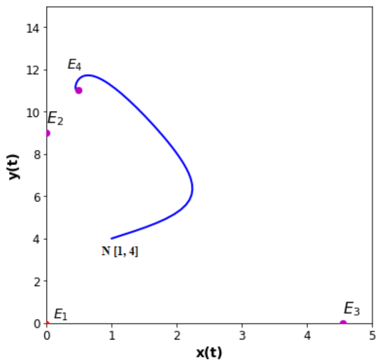

Figure 2. Phase Portraits of The System (1) With e=2.1 And Initial Value [1,4] Convergent To E_4.

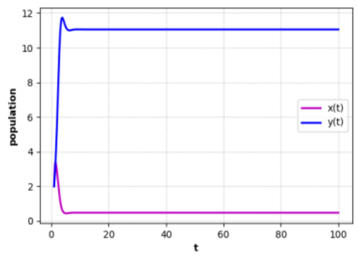

Figure 4. Time Series of The System (1) With e=2.1 And Initial Value [2,2]. The Blue Curve Represents the Predator Population, and the Magenta Curve Represents the Prey Population.

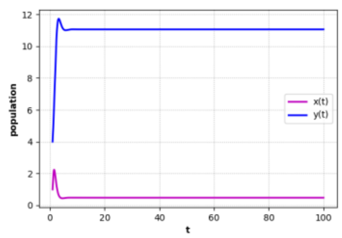

Figure 5. Time Series of The System (1) With e=2.1 And Initial Value [1,4]. The Magenta and Blue Curve Represents the Prey and Predator Population

In Figure 2, when using the initial value [2,2] the system solution converges to the equilibrium point $E_4 = (0.48,11.05)$. In Figure 3, when using the initial value [1,4] the system solution also converges to the equilibrium point $E_4 = (0.48,11.05)$. So, it is clear that the equilibrium point. E_4 is asymptotically stable, and the solution graphs shown in the phase portraits in Figure 2 and Figure 3 show the equilibrium point E_4 is asymptotically stable. Therefore, it is concluded that the system solution always converges to the equilibrium point E_4. This shows the agreement between the simulation results and analysis and can be interpreted that the predator population and prey population exist and can coexist.

2. The second simulation with environmental carrying capacity parameter $e = 2.878$

   Based on the parameter values used in Table 2 with $e = 2.878$ three equilibrium points exist, namely. $E_1 = (0,0), E_2 = (0,12.3), E_3 = (4.6,0)$. Analysis of stability with these parameter values, the eigenvalues of each equilibrium point are obtained as follows:

   a. $E_1 = (0,0)$, with $\lambda_1 = 1.2 > 0 \wedge \lambda_2 = 7.4 > 0$, equilibrium point E_1 is unstable.
   b. Equilibrium point $E_2 = (0,12.3)$ is asymptotically stable with $\lambda_1 = -1.2 < 0 \wedge \lambda_2 = -0.0006 < 0$, if condition $e > \frac{m(b+c_1)}{ar}$
   c. Equilibrium point $E_3 = (4.6,0)$ is unstable with $\lambda_1 = -7.4 < 0 \wedge \lambda_2 = 1.2 > 0$.

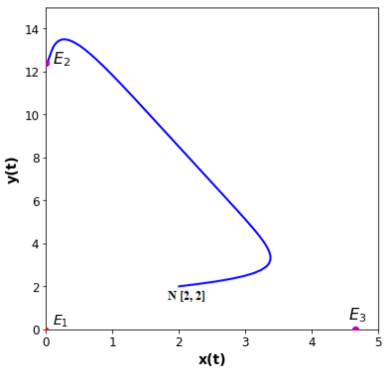

Figure 6. Phase Portraits of The System (1) With e=2.878 And an Initial Value [2, 2]

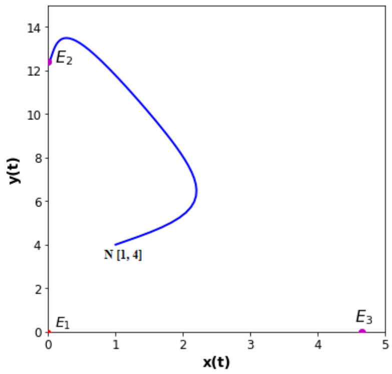

Figure 7. Phase Portraits of The System (1) With e=2.878 And an Initial Value [1,4]

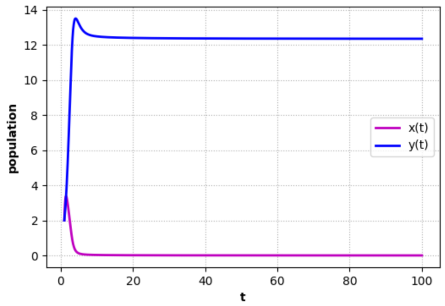

Figure 8. Time Series of The System (1) With e=2.878 And an Initial Value [2,2] The Magenta and Blue Curve Represents the Prey and Predator Population

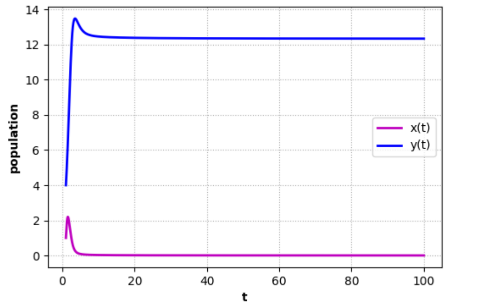

Figure 9. Time Series of The System (1) With e=2.878 And an Initial Value [1,4] The Magenta and Blue Curve Represents the Prey and Predator Population

Increasing the environmental carrying capacity parameter to $e = 2.878$ only the equilibrium point E_2 is asymptotically stable. In Figure 6, when using the initial value [2,2] the system solution converges to the equilibrium point $E_2 = (0,12.3)$. In Figure 7, when using the initial value [1,4] the system solution also converges to the equilibrium point $E_2 = (0,12.3)$. So, it is clear that the equilibrium point. E_2 is asymptotically stable, and the solution graphs shown in the phase portraits in Figure 6 and Figure 7 show the equilibrium point E_2 is asymptotically stable. Therefore, it is concluded that the system solution always converges to the equilibrium point E_2. This shows that when the environmental carrying capacity parameter of the predator population is increased, it results in the extinction of the prey population.

Conclusion

This research discusses the *predator-prey* model by considering the cannibalism behaviors of the prey. There are four equilibrium points obtained: $E_1 = (0,0)$, $E_2 = \left(0, \frac{re}{m}\right)$, $E_3 = \left(\frac{b+c_1}{c+1}, 0\right)$, and $E_4 = \left(-\frac{aer-bm-c_1m}{ar+cm+m}, \frac{r(ce+b+c_1+e)}{ar+cm+m}\right)$ with the condition that $e < \frac{m(b+c_1)}{ar}$. The local stability analysis of the system, it is concluded that the equilibrium point E_1 and E_3 are unstable. The stability of the equilibrium point E_2 is asymptotically stable if $e > \frac{m(b+c_1)}{ar}$. Then, an equilibrium point E_4 is asymptotically stable if $e < \frac{ar^2+bcm+cc_1m+cmr+bm+c_1m+mr}{ar(c+1)}$ and $\frac{m(abr+ac_1r+bcm+bm+cc_1m+c_1m)}{r(a^2r+acm+amr)} < e$. Numerical simulations show that only the equilibrium point E_4 which is asymptotically stable when the environment carrying capacity parameter $e = 2.1$ that both populations exist or can coexist. Meanwhile, when $e = 2.878$ then, only E_2 is asymptotically stable, which states that the population of prey population is extinct. Based on the numerical simulation results, the environmental carrying capacity parameter affects the change of the system solution. In this research, also using two different initial values, it is concluded that the system solution always converges to the equilibrium point E_2 and E_4.

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