

## Sensitivity Analysis of Online Game Addiction Model

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### ABSTRACT

*The rapid advancement of the internet has led to some shifts in daily activities. Some conventional activity is now slowly shifted to online, including playing online games. Online games have both impacts, positive and negative. The advantage of online games is to help people communicate with other parties anywhere and anytime in the world. On the other hand, the negative impact is an addiction which leads to losing track of time and neglecting their responsibilities. This study aims to analyse online game addiction using a mathematical model in a population. The mathematical model used is the SEAR model with four compartments: susceptible, exposed, addiction and recovery. The population of this study is limited to cases of addiction to online games on gadgets and ignores the factor of having professional players (e-sports). The data used in this research is students from Faculty of Mathematics and Natural Sciences Unesa. We obtained the basic reproduction number is  $R_0 = 0.836$ , which means that online game addiction do not occurs in the population. Sensitivity analysis showed that the rate of susceptible gamer to be exposed gamer ( $\alpha$ ) and the rate of individuals who addicted to online game ( $\beta$ ) are the sensitive parameter. The numerical simulation also shown that the change in  $\alpha$  brings an impact to the overall dynamics.*

**Keywords:** online games, addiction, sensitivity analysis

### Introduction

In recent decades, online gaming is getting more popular. Recent studies in the UK shows that 88% of respondent aged 16-24 are playing video games, of which 29% were playing online with anonymous people worldwide [1]. The emergence of gamers happened almost everywhere in the world, including Indonesia. From the online gaming survey conducted by Rakuten Insight in April 2022, nearly 50% of respondents play online games daily, and respondents who rarely play online games took the portion of 6% [2].

The massive penetration of online games also brings advantages and disadvantages. The benefit which is seen everywhere is the emergence of the e-sport. E-sport has been included in one field that competed in Asian Games 2018 Jakarta-Palembang [3], though it hasn't been admitted to the Olympic games. Indonesia has already made the e-Sport official, although it is still debatable [4]. Other than that, online games allow the younger generation to interact with anyone worldwide. The disadvantages, online games make people getting addicted. World Health Organisation considered Internet Gaming Disorder (IGD) as a mental disorder by the 11th Revision of the International Classification of Diseases (ICD-11) as a pattern of gaming behaviour ("digital-gaming" or "video-gaming") [5]. Someone is considered to suffer IGD if they show increasing priority to gaming over other activities, gaming takes over other daily activities, and escalation of gaming despite the occurrence of negative consequences [6]. Li & Guo stated that based on the intensity of playing online games, it could be categorised into three major parts which are healthy game which spends three hours per day, fatigue game which spends three to five hours per day, and the last, unhealthy game which spends more than five hours per day [7].

Many studies have been conducted to find solutions to this addiction problem. Addiction problem is not only occurred in online games, but also in some other extent such as addiction in alcoholic beverages which are studied by Abrori in 2016 [8]. Indah & Maulana also studied the addiction in social media, they took a case study of tiktok addiction in students of FMIPA Unesa [9]. Addiction in smokers also studied by Fadhilah & Maulana. They divide the population based on gender, the male smokers and the female smokers [10]. Research by Novrialdy in 2019 studied the impact of online gaming addiction on teenagers and how to overcome it [11]. Utami et al. in 2020 also examined the relation between game addiction and the social adjustment ability in adolescents [12]. Not only researchers in social science but some mathematicians also researched to study online gaming addiction. Li & Guo in 2019 investigated the stability and optimal control of online game addiction [7] They categorised the population into four compartments: Susceptible, Infective, Professional and Quitting. Anwar et al. modelled the SEIRS model to model online game addiction [13]. Later in 2022, Seno studied the mathematical population dynamics model for internet gaming addiction [14]. He modelled the population into three compartments: moderate, addictive and recovery/under treatment. In the same year, Surya & Maulana modelled the dynamics of online game addiction in personal gadgets [15].

## Research Methods

This research is quantitative research which conducted with a numerical simulation to obtain the sensitive parameter(s). The data used in this research is the primary data which obtained from a survey in Faculty of Mathematics and Natural Sciences, Universitas Negeri Surabaya. The research commences with constructing a model corresponding to the online game addiction, then the equilibria is examined and analysed. After that, by plugging in some parameters obtained from the survey, the numerical simulation is performed. The parameters are then further analysed to obtain the sensitive parameters. Therefore, the sensitive parameters are obtained and the conclusion is drawn.

## Results and Discussion

To construct the mathematical model, we set some assumptions:

- Population is divided into four subpopulation which are Susceptible ( $S$ ), Exposed ( $E$ ), Addicted ( $A$ ) and Recovery ( $R$ ).
- Susceptible is the subpopulation in which people have played online games not in their personal gadgets, they don't have any games in their personal gadget.
- Exposed is the subpopulation in which people have installed online games in their personal gadget yet they play less than five hours per day.
- Addicted is the subpopulation in which people have installed online games in their personal gadget yet they play more than five hours per day.
- Recovered is the subpopulation in which people have uninstalled online games in their personal gadget or stop to play online games.
- Individuals in  $S$  have the potential to move to  $E$  upon interacting with people who are addicted to online games, so does with individuals in  $E$ . They have the potential to move to  $A$  upon interactions.
- Individuals in  $S$  has only ever played online games on other people's devices and not their own, it can also be assumed that they play online games for under five hours per day and only play online games a few times.
- Individuals in  $S$  cannot move directly to  $A$  because addiction needs time to grow.
- Individuals in  $E$  can do both behaviour: they can be addicted to game, or they can be recovered from the pre-addiction.

### Mathematical Model

From those assumption, we can obtain a compartment model as follow:

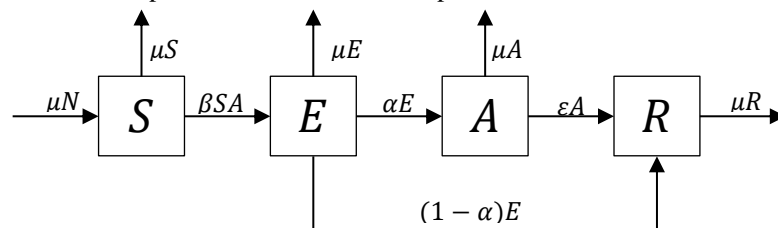


Figure 1. Compartment model of addiction model

Based on the compartment above, we can obtain a mathematical model as follow:

$$\begin{aligned}
 \frac{dS}{dt} &= \mu N - \beta SA - \mu S \\
 \frac{dE}{dt} &= \beta SA - \alpha E - (1 - \alpha)E - \mu E \\
 \frac{dA}{dt} &= \alpha E - \varepsilon A - \mu A \\
 \frac{dR}{dt} &= \varepsilon A + (1 - \alpha)E - \mu R
 \end{aligned} \tag{1}$$

Where:

Table 1. Parameters

Parameter	Description	Requirement	Value
$\mu$	The rate of new gamer and quit gamer	$\mu > 0$	0.102
$\beta$	The rate of susceptible gamer to be exposed gamer	$\beta > 0$	0.28
$\alpha$	The rate of individuals who addicted to online game	$\alpha > 0$	0.5
$\varepsilon$	The rate of individuals who quit from the addiction	$\varepsilon > 0$	0.831

### Equilibria

We obtain two equilibria: the addiction-free equilibrium  $E_0 = (S^*, E^*, A^*, R^*) = (1, 0, 0, 0)$  and endemic equilibrium  $E_1 = (S^{**}, E^{**}, A^{**}, R^{**})$ , where: [1]–[3]

$$S^{**} = \frac{\varepsilon\mu + \mu^2 + \varepsilon + \mu}{\alpha\beta}$$

$$E^{**} = \frac{\mu(\alpha\beta - \varepsilon\mu - \mu^2 - \varepsilon - \mu)}{\alpha(\mu + 1)\beta}$$

$$A^{**} = \frac{\mu(\alpha\beta - \varepsilon\mu - \mu^2 - \varepsilon - \mu)}{\beta(\varepsilon\mu + \mu^2 + \varepsilon + \mu)}$$

$$R^{**} = -\frac{(\alpha\mu - \varepsilon - \mu)(\alpha\beta - \varepsilon\mu - \mu^2 - \varepsilon - \mu)}{\alpha(\mu + 1)(\varepsilon + \mu)\beta}$$

### Basic Reproduction Number

By using the Next-Generation Matrix and solving the equation  $\det(\lambda I - FV^{-1}) = 0$  we obtain the basic reproduction number

$$R_0 = \frac{\beta\alpha}{(\mu + \varepsilon)(\mu + 1)} = 0.836 \quad (2)$$

When  $R_0 < 1$  the system is said to be asymptotically stable to the addiction-free equilibrium. It means that the individuals who are addicted to online game will be removed from the population. On the other hand, when  $R_0 > 1$  the system is said to be asymptotically stable to the endemic equilibrium, which means the individuals who are addicted to online gaming will always exist and increase.

Since  $0.836 < 1$ , the system is free from addiction. Individuals addicted to online games are removed from the population at some point.

### Stability Analysis

#### Addiction-free equilibrium

By performing a linearization at  $E_0 = (S^*, E^*, A^*, R^*) = (1, 0, 0, 0)$ , we obtain the Jacobian matrix as follow:

$$J(E_0) = \begin{bmatrix} -\mu & 0 & -\beta & 0 \\ 0 & -1 - \mu & \beta & 0 \\ 0 & \alpha & -(\varepsilon + \mu) & 0 \\ 0 & 1 - \alpha & \varepsilon & -\mu \end{bmatrix} \quad (3)$$

Solving the characteristic equation (3), we obtain

$$(\lambda + \mu)(-\alpha\beta\lambda - \alpha\beta\mu + \varepsilon\lambda^2 + 2\varepsilon\lambda\mu + \varepsilon\mu^2 + \lambda^3 + 3\lambda^2\mu + 3\lambda\mu^2 + \mu^3 + \varepsilon\lambda + \varepsilon\mu + \lambda^2 + 2\lambda\mu + \mu^2) = 0 \quad (4)$$

And the eigenvalues:

$$\lambda_1 = -\mu = -0.102$$

$$\lambda_2 = -\frac{1}{2}\varepsilon - \mu - \frac{1}{2} + \frac{1}{2}\sqrt{4\alpha\beta + \varepsilon^2 - 2\varepsilon + 1} = -0.634$$

$$\lambda_3 = -\frac{1}{2}\varepsilon - \mu - \frac{1}{2} - \frac{1}{2}\sqrt{4\alpha\beta + \varepsilon^2 - 2\varepsilon + 1} = -1.401$$

are all negative. Therefore, the addiction-free equilibrium  $E_0$  will asymptotically stable when

$$4\alpha\beta + \varepsilon^2 > 2\varepsilon + 1$$

$$\varepsilon + 2\mu + 1 > \sqrt{4\alpha\beta + \varepsilon^2 - 2\varepsilon + 1}$$

### Endemic equilibrium

Repeating the same process for  $E_1 = (S^{**}, E^{**}, A^{**}, R^{**})$ , we obtain the Jacobian matrix

$$J(E_1) = \begin{bmatrix} -\frac{\mu(\alpha\beta - \varepsilon\mu - \mu^2 - \varepsilon - \mu)}{(\varepsilon\mu + \mu^2 + \varepsilon + \mu)} - \mu & 0 & -\frac{\varepsilon\mu + \mu^2 + \varepsilon + \mu}{\alpha} & 0 \\ \frac{\mu(\alpha\beta - \varepsilon\mu - \mu^2 - \varepsilon - \mu)}{(\varepsilon\mu + \mu^2 + \varepsilon + \mu)} & -1 - \mu & \frac{\varepsilon\mu + \mu^2 + \varepsilon + \mu}{\alpha} & 0 \\ 0 & \alpha & -(\varepsilon + \mu) & 0 \\ 0 & 1 - \alpha & \varepsilon & -\mu \end{bmatrix} \quad (5)$$

Solving the characteristic equation (5), we obtain

$$\frac{1}{(1 + \mu)(\varepsilon + \mu)} \left( (\mu^5 + (2\varepsilon + 2)\mu^4 + (-\alpha\beta + \varepsilon^2 + 2\lambda^2 + 4\varepsilon + 1)\mu^3 \right. \\ \left. + (\lambda^3 + (3\varepsilon + 3)\lambda^2 + 2\alpha\beta\lambda - (\varepsilon + 1)(\alpha\beta - 2\varepsilon))\mu^2 \right. \\ \left. + ((\varepsilon + 1)\lambda^3 + (\alpha\beta + \varepsilon^2 + 4\varepsilon + 1)\lambda^2 + \beta\alpha(\varepsilon + 1)\lambda - \beta\alpha\varepsilon + \varepsilon^2)\mu \right. \\ \left. + \lambda^2\varepsilon(\lambda + \varepsilon + 1) \right) (\lambda + \mu) = 0 \quad (6)$$

By substituting the parameters into the equation (6), we obtain the eigenvalues:

$$\lambda_1 = -0.102, \quad \lambda_2 = 0.0038 + 0.2098i, \quad \lambda_3 = -2.057, \quad \lambda_4 = 0.0038 - 0.2098i$$

### Sensitivity Analysis

Sensitivity analysis investigates some parameters' impact on the basic reproduction number  $R_0$  and later impact of such parameters on the addiction model. To perform the analysis, we calculate the normalized forward sensitivity index of  $R_0$ . The normalized forward sensitivity index of a function,  $F(x_1, x_2, \dots, x_n)$ , for  $x_i (1 \leq i \leq n)$ , is defined as:

$$\Gamma_{x_i}^F = \frac{\partial F}{\partial x_i} \times \frac{x_i}{F}$$

To find the sensitivity of  $R_0$ , we consider the parameters  $\alpha, \beta, \mu$ , and  $\varepsilon$  as  $R_0$  is the function of these parameters. We obtain:

Table 2. Sensitivity Index

Parameters	$\alpha$	$\beta$	$\mu$	$\varepsilon$
Sensitivity index	1	1	-0.764	-0.329

From the sensitivity index, it can be shown that parameters  $\alpha$  and  $\beta$  are positive. It means that an increment of 10% in one of the parameters, either  $\alpha$  or  $\beta$  will cause  $R_0$  increased by 10%, while an increment of 10% in  $\mu$  and  $\varepsilon$  will cause  $R_0$  decreased by 7.64% and 3.29% respectively. We can see that the most sensitive parameters positively impact  $R_0$ . The rate of individuals addicted to online games ( $\alpha$ ) and the rate of susceptible gamers to be exposed gamers ( $\beta$ ).

### Numerical Simulations

Setting up the initial values of each population  $S(0) = 49, E(0) = 140, A(0) = 24, R(0) = 9$  and simulate it for 21 days.

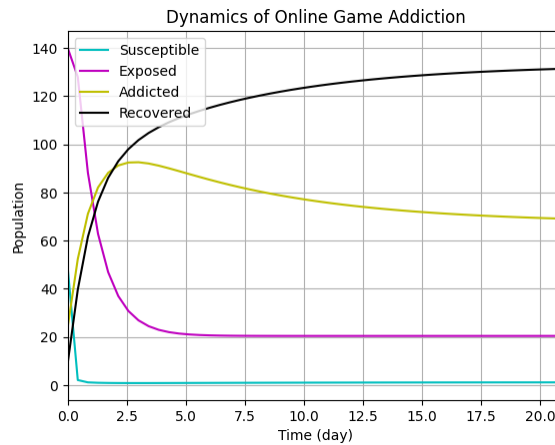


Figure 2. The Dynamics of Online Game Addiction

Figure 1 shows a significant decrease in susceptible and exposed and a meaningful increase in addicted and recovered. As the basic reproduction number is less than 1, it clearly shows that the individuals who addicted to online games will converge to a certain number, in this case, 70. The initial population in Exposed was 140 then drastically decreased to just 20 in just six days, to increase in addiction and recover. Individuals who are addicted to online games were just 24 initially. During the first two days, an additional individual is admitted to this category. As for the recovered, the individuals who recovered are increased before it stabilizes in the end. By plotting the sensitive parameters, we plot the graph of  $\alpha$  to show the impact on  $A$  as shown in figure 2.

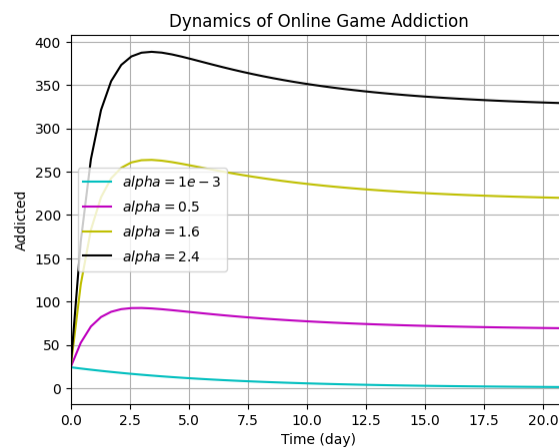


Figure 3. Different values of  $\alpha$  on  $A$

It can be showed that the smaller the value of  $\alpha$ , the individuals who addicted to online games tend to converge to zero, and vice versa. Then, we plot the complete dynamics using the different values of  $\alpha$ .

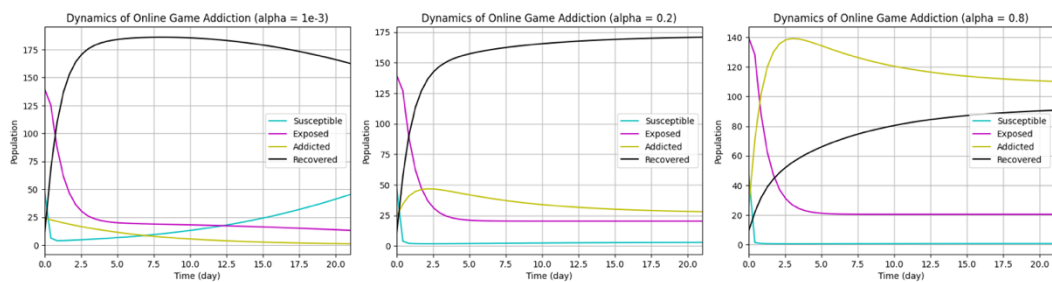


Figure 4. The Impact of change in  $\alpha$  on Addiction Model Dynamics

It depicts those different values of  $\alpha$  plays an important role in the model dynamics. The latter figure show that if we plug  $\alpha = 0.8$ , the individuals who addicted to online games exceed the individuals who quit playing.

### Conclusion

Based on the result and discussion, we obtain the two equilibria: the addiction-free equilibrium  $\lambda_1 = -0.102$ ,  $\lambda_2 = -0.634$ ,  $\lambda_3 = -1.401$  and endemic equilibrium  $\lambda_1 = -0.102$ ,  $\lambda_2 = 0.0038 + 0.2098i$ ,  $\lambda_3 = -2.057$ ,  $\lambda_4 = 0.0038 - 0.2098i$ . We also obtain the basic reproduction number  $R_0 = 0.836$  which means that the population is free from addiction. In other words, there is no online games addiction in the population. Sensitivity analysis also shows that the sensitive parameters are  $\alpha$  and  $\beta$ . The numerical simulation also shown that the change in  $\alpha$  brings an impact to the overall dynamics.

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