

## Analysis of Students' Errors in Solving Word Problems on Probability Material for Grade VIII based on Newman's Stages in Terms of Mathematical Ability Levels

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**ABSTRACT.** The purpose of this study is to describe the types of errors that arise when students solve probability problems by applying Newman's stages, viewed from students' levels of mathematical ability. This descriptive study employed a qualitative approach. The research subjects consisted of six eighth-grade students from MTs Al Huda. The subjects were selected using purposive sampling techniques. The instruments used in this study included a written test and an interview guide. The written test consisted of two items, and the interview guide was validated by experts and declared valid. Students with high mathematical ability made errors at the process skill stage. The leading cause of errors among students with high mathematical ability was poor calculation accuracy. Meanwhile, students with moderate mathematical ability made errors at the transformation and process skill stages. The primary factors causing errors among students with moderate mathematical ability were a lack of understanding of changes in the sample space in situations without replacement and insufficient understanding of fraction simplification rules. On the other hand, students with low mathematical ability experienced errors at the reading, transformation, process skill, and encoding stages. Several significant factors contributing to errors made by students in the low-ability category include: (1) errors in interpreting words or terms in the problems, (2) a lack of understanding of probability concepts, (3) insufficient understanding of fraction simplification rules, and (4) inability to conclude the solutions in accordance with the given problems.

**Keywords:** error analysis; mathematical ability level; probability

**ABSTRAK.** Tujuan penelitian ini adalah untuk mendeskripsikan kesalahan-kesalahan yang muncul ketika siswa mengerjakan soal tentang peluang dengan menerapkan tahapan Newman, dengan meninjau dari tingkat kemampuan matematika siswa. Penelitian deskriptif ini menggunakan pendekatan kualitatif. Subjek penelitian merupakan enam siswa dari MTs Al Huda kelas 8. Subjek dipilih dengan menggunakan teknik *purposive sampling*. Dalam penelitian ini, instrumen yang digunakan meliputi tes tulis dan pedoman wawancara. Tes tulis terdiri atas dua butir dan pedoman wawancara telah divalidasi oleh ahli dan dinyatakan valid. Siswa dengan kemampuan matematika tinggi melakukan kesalahan pada tahapan *process skill*. Penyebab utama terjadinya kesalahan pada siswa dengan kemampuan matematika tinggi adalah kurangnya ketelitian pada saat melakukan perhitungan. Sementara itu, siswa dengan kemampuan matematika sedang melakukan kesalahan pada tahap *transformation* dan *process skill*. Faktor utama yang menyebabkan kesalahan pada siswa dengan kemampuan matematika sedang yakni kurangnya pemahaman terhadap perubahan ruang sampel dalam situasi tanpa pengembalian dan kurangnya pemahaman terhadap aturan penyederhanaan pecahan. Di sisi lain, siswa dengan kemampuan matematika rendah mengalami kesalahan pada tahap *reading*, *transformation*, *process skill*, hingga *encoding*. Beberapa faktor utama penyebab kesalahan yang dilakukan oleh siswa dengan kategori kemampuan rendah: (1) kesalahan dalam memaknai kata atau istilah pada soal, (2) kurangnya pemahaman konsep peluang, (3) kurangnya pemahaman terhadap aturan penyederhanaan pecahan, dan (4) tidak dapat menentukan kesimpulan dari jawaban sesuai dengan permasalahan yang diberikan.

**Kata kunci:** analisis kesalahan; tingkat kemampuan matematika; peluang

## INTRODUCTION

Mathematics plays a vital role in human life because it trains individuals to think systematically, logically, and critically, and helps in solving real-world problems (Yunaeti, Arhasy, & Ratnaningsih, 2021). In mathematics and other scientific fields, these thinking skills are helpful for decision-making and problem-solving (Mutmainah, Hermawati, & Maulana, 2019). Mathematical problem solving in schools is generally taught through word problems. Students' ability to handle word problems, which is closely related to problem-solving skills, has significant benefits in everyday life (Utari, Wardana, & Damayani, 2019). Solving word problems involves the ability to resolve problems that represent real-life situations (Possumah & Rofiki, 2024).

One important area of mathematical problem solving in everyday life is probability, which is often presented in the form of word problems (Muslim, Prayitno, Salsabila, & Amrullah, 2022). Probability is a crucial topic in mathematics education because it helps students better understand uncertainty (Batanero, Chernoff, Engel, Lee, & Sanchez, 2016). In addition, Indriani (2019) states that probability also helps students develop other mathematical abilities, such as problem-solving skills, critical thinking, and logical reasoning. Learning probability enables students to deal with changing situations, such as predicting the success of medical treatments, evaluating life expectancy estimates, reducing the risk of losses, and understanding weather forecasts (Rada & Fauzan, 2019). By understanding and mastering probability, students will be better prepared to make informed decisions in everyday situations involving uncertainty.

Furthermore, empirical and theoretical probability are included in Basic Competency 4.11 of the 2013 Curriculum, which emphasizes students' ability to solve such problems (Kemendikbud, 2018). In addition, the Learning Outcomes in the Merdeka Curriculum Phase D (junior high school or Package B) expect students to be able to explain and apply the concepts of probability and relative frequency (also known as empirical probability) to determine the expected frequency of an event in simple experiments (Kemendikbud, 2022). Therefore, understanding probability is not only useful in everyday life but also an important part of the educational curriculum. One way to determine whether the objectives of mathematics learning have been achieved is to examine students' mastery of the learning outcomes (Yunita, Juwita, Suci, & Kartika, 2020). If students experience difficulties in mastering the material, the learning objectives are not achieved, and consequently, the curriculum's basic competencies are not fully attained.

However, in reality, students often experience difficulties understanding probability concepts, including identifying the sample space and sample points, and distinguishing between theoretical and empirical probability (Mutiarahman, Edriati, & Suryani, 2023). These difficulties are reflected in students' errors when solving problems (Ni'mah, Sunismi, & Fathani, 2018). This is supported by the observation results, which showed that 9 out of 15 eighth-grade students at MTs Al Huda made errors when solving probability word problems. The problem involved calculating probability in a marble-drawing scenario. The problem stated that a bag contained five red marbles and two blue marbles. On the first draw, a red marble was drawn and not replaced. Students were then asked to determine the probability of drawing a red marble on the second draw.

The researcher presents an example of an error made by student DN, as shown in Figure 1. Based on the written answer, student DN incorrectly determined the sample space after drawing the first marble. Because one marble was drawn without being returned, the number of marbles in the bag decreased.

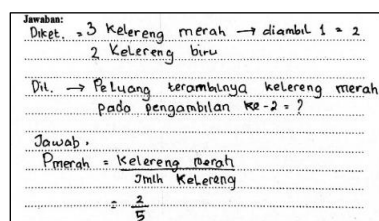


Figure 1. DN Student Work Results

In the interview, DN argued that this reduction had no impact on the sample space. In fact, the sample space for the second draw is the remaining four marbles in the bag. Therefore, the probability of drawing a red marble on the next draw is  $\frac{2}{4}$ , not  $\frac{2}{5}$ .

Given the errors that occurred, it is important to investigate the material's potential for further mistakes. One helpful approach to investigating errors is Newman's Theory, better known as Newman's Error Analysis (NEA). Newman states that a student must be able to progress through several successive stages: Reading, Comprehension, Transformation, Process Skills, and Encoding (White, 2018). Newman's stages have also been widely recognized as having the highest level of credibility compared to other procedures (Restuningsih & Khabibah, 2021). Thus, Newman's stages not only help identify students' errors but also indicate where those errors occur.

Students' mathematical ability is one of the factors influencing their ability to solve mathematical problems (Isroil, Budayasa, & Masriyah, 2017). Students' mathematical ability is their capacity to solve mathematical problems. Mathematical ability can be determined based on student learning outcomes (Susilawati, Sridana, Novitasari, & Subarinah, 2023). Mathematical ability can be classified into three categories: high, medium, and low. By considering mathematical ability, potential errors at each level can be identified (Siswandi, 2021).

Recently, many relevant studies have analyzed student errors in probability using Newman's stages. The results of Tanzimah & Sutrianti (2023) study indicate the percentages for each error type: 61.90% for reading errors, 23.80% for comprehension errors, 55.95% for transformation errors, 53.57% for process skills errors, and 67.85% for final answer writing errors. Furthermore, research by Muslim, Prayitno, Salsabila, & Amrullah (2022) also analyzed student errors in probability based on learning styles, and research by Salamah & Amelia (2020) examined gender and self-confidence. However, research on student errors by mathematical ability level is still rare. Therefore, the purpose of this study is to describe the errors students make when working on probability problems using Newman's stages. The difference between this study and previous studies lies in the review based on mathematical ability levels. It is hoped that this research can contribute to error analysis in probability. By understanding students' errors and the stage at which they occur, teachers can provide more appropriate and practical guidance and intervention.

## METHOD

This descriptive research uses a qualitative approach to explore and deeply understand events experienced by individuals or groups related to social or humanitarian issues. The qualitative research process involves conducting interviews, collecting data, analyzing it, and interpreting its meaning (Creswell, 2014).

Six students from MTs Al Huda served as research subjects. Two students were selected for each math ability category. These categories were determined based on the average Mid-Semester Assessment (PTS) and Final-Semester Assessment (PAS) math scores for the odd semester of the 2023/2024 academic year, as outlined in Table 1.

**Table 1. Student Ability Level**

| Number | Score Interval      | Level of Mathematical Ability |
|--------|---------------------|-------------------------------|
| 1      | $x < 40$            | Low                           |
| 2      | $41 \leq x \leq 70$ | Medium                        |
| 3      | $x > 70$            | High                          |

A purposive sampling technique was used in the subject selection process, selecting students with the highest, middle, and lowest scores to prevent bias or tendencies in mathematical ability. Furthermore, the research subjects were willing and agreed to participate. The categorization of students' mathematical skills yielded six research subjects, as listed in Table 2. The selected subjects

included T1 and T2 for high levels of mathematical ability, S1 and S2 for medium levels, and R1 and R2 for low levels.

**Table 2. Characteristics of Research Subjects**

| Number | Name | Value PTS | Value PAS | Mean  | Category | Selected Subjects |
|--------|------|-----------|-----------|-------|----------|-------------------|
| 1      | GAK  | 97        | 93.5      | 95.25 | High     | T1                |
| 2      | ZQ   | 72        | 77.5      | 74.75 | High     | T2                |
| 3      | AHA  | 60        | 61.5      | 60.75 | Medium   | S1                |
| 4      | MPFC | 59        | 62        | 60.5  | Medium   | S2                |
| 5      | FEA  | 33        | 33        | 33    | Low      | R1                |
| 6      | PRA  | 29        | 22        | 25.5  | Low      | R2                |

The instruments used in this study included a written test and an interview guide. The written test, consisting of two questions, and the interview guide were validated by a mathematics education expert with at least 5 years of teaching experience and a master's degree in mathematics education. Both instruments were declared valid by the validator and suitable for use in the study. The descriptive questions completed by the students are presented below: 1) Diana has a bag containing nine balls, consisting of 4 red balls and five blue balls. Diana takes a ball from the bag without looking. The ball turns out to be blue and is not returned to the bag. If the blue ball she took is not returned to the bag, what is the probability of drawing a blue ball on the next draw? 2) A child experiments by rolling a die. The result is 6 times a 1, 12 times a 2, 11 times a 3, 8 times a 4, 10 times a 5, and 13 times a 6. Determine the empirical probability of rolling a prime number.

To obtain more comprehensive or in-depth data, researchers used triangulation. Triangulation is the process of gathering information from multiple sources and examining the data thoroughly to strengthen research findings (Creswell, 2014). In this study, triangulation was achieved by combining the results of written tests and interviews, a process known as method triangulation.

The data analysis techniques used include data reduction, data display, and conclusion drawing (Miles & Huberman, 1994). In analyzing errors, the test and interview results were adjusted to Newman's error indicators, as presented in Table 3.

**Table 3. Newman Error Indicator**

| Number | Newman Stages        | Error Indicator  |
|--------|----------------------|--|
| 1      | Reading error        | a. Students are not accurate in reading words, symbols, or units.<br>b. Students are unable to understand the meaning of each word, term or symbol in the question.  |
| 2      | Comprehension error  | a. The student did not write down the known information from the question.<br>b. The student did not write down the known information in the question accurately.<br>c. The student did not write down the information asked for in the question.<br>d. The student did not write down the information asked for in the question accurately.<br>e. The student used unclear symbols when writing down the known information. |
| 3      | Transformation error | a. The student is unable to convert the problem into mathematical form.<br>b. The student is unable to convert the problem into mathematical form correctly.<br>c. The student is unable to determine the formula used to solve the problem correctly.   |
| 4      | Process skill error  | a. The student made an error in the calculation.<br>b. The student did not continue the calculation when solving the problem.  |
| 5      | Encoding error       | a. The student did not write the notation correctly.<br>b. The student did not write the conclusion of the answer.<br>c. The student did not correctly determine the conclusion of the answer.   |

Source: Modification of Dea & Anik (2020) dan Dewi & Kartini (2021).

## RESULTS AND DISCUSSION

### Student Errors in the High Mathematics Ability Category

#### *Higher Mathematics Ability Subject T1*

When completing question 2, Subject T1 correctly interpreted the question and completed the reading stage. Furthermore, Subject T1 was able to identify information and correctly understand the question, enabling him to complete the comprehension stage successfully. Furthermore, Subject T1 determined the correct solution formula to obtain the requested answer, allowing him to complete the transformation stage successfully. However, during the process skill stage, errors were recorded in Figure 2.

$$P(A) = \frac{f(A)}{n} \rightarrow \frac{12+11+10}{60} = \frac{33}{60} = \frac{11}{30}$$

Figure 2. Subject T1's Answers

In the process skills stage, Subject T1 made an error in mathematical calculations. Subject T1's error occurred when simplifying the fraction  $33/60$  to  $11/30$ . This error occurred because Subject T1 divided the numerator (33) and denominator (60) by 3, believing the result was  $11/30$ . The result should have been  $11/20$ , not  $11/30$ , as stated by Subject T1. This is evidenced by the following excerpt from an interview.

*Q : Okay. Are you sure about that answer?*

*T1 : Sure, sis.*

*Q : Where does  $11/30$  come from?*

*T1 : I divided 33 by 3, then I divided 60 by 3, which makes  $11/30$ , sis. Oh, sorry, I wasn't careful enough. 60 divided by 3 equals 20. So, the result is  $11/20$ , sis.*

Based on the interview, Subject T1 realized his error after the researcher asked him to explain the simplification process. This awareness indicates that Subject T1 acknowledged his mistake, which resulted from carelessness in his calculations. However, after realizing his error, Subject T1 provided the correct answer. In the encoding stage, the student stated a conclusion that aligned with the final result of the process skill stage. Therefore, Subject T1 completed the encoding stage correctly.

#### *Higher Mathematics Ability Subject T2*

When completing the problem, Subject T2 correctly interpreted the problem, thus completing the reading stage. Furthermore, Subject T2 was able to identify information and correctly understand the questions, thereby completing the comprehension stage. Furthermore, Subject T2 determined the correct solution formula to obtain the requested answer, thus completing the transformation stage correctly. The calculations performed by Subject T2 also showed the proper method and results, thus completing the process skill stage correctly. At the encoding stage, Subject T2 stated a conclusion in accordance with the final result at the process skill stage. Therefore, Subject T2 completed the encoding stage correctly. Subject T2's written answer is presented in Figure 3.

$DH = \text{Dadu 1} = 6 \times$   
 $-11 - 2 = 12 \times$   
 $-11 - 3 = 11 \times$   
 $-11 - 4 = 8 \times$   
 $-11 - 5 = 10 \times$   
 $-11 - 6 = 13 \times$   
 Dik = peluang muncul mata dadu bil. prima  
 $Jwb = \frac{P(A)}{n}$   
 $P(\text{bil. prima}) = \frac{23}{60} = 11$   
 Jadi peluang empiris muncul mata dadu bil. prima adlh  $\frac{11}{20}$

Figure 3. Subject T2's Answers

### Student Errors in the Moderate Mathematics Ability Category

#### Subject of Medium Mathematics Ability S1

When completing question 1, Subject S1 correctly interpreted the question, enabling him to complete the reading stage. Furthermore, Subject S1 was able to identify information and understand the question correctly, allowing him to complete the comprehension stage correctly. However, Subject S1 made an error in determining the solution procedure and did not include a conclusion on his answer sheet. This is shown in Figure 4.

$Jwb = P(B) = \frac{n(B)}{n(S)} = \frac{4}{9}$   
 <Balabiru:  $5 - 1 = 4$ >  $\frac{n(B)}{n(S)}$

Figure 4. Subject S1's Answers

On the answer sheet, Subject S1 wrote that the number of elements in his sample space, or  $n(S)$ , was the initial number of balls, namely 9. Subject S1 also assumed that drawing a ball without replacement did not affect the sample space, even though the sample space should have decreased after the first drawing, so that  $n(S)$  on the second drawing was 8. The following excerpt from interview evidence this.

*Q : Here, you mention that Diana took the blue ball from the bag and did not replace it. Does this affect the sample space?*

*S1 : No*

*Q : Why did you state that the sample space was 9?*

*S1 : Because the number of balls was 9.*

In the interview, Subject S1 explained that he initially wrote  $n(S)$  as the number of balls, which totaled 9. This indicates that he did not understand the concept of the change in the sample space that should occur after the first drawing, leading Subject S1 to make an error in the transformation stage.

Transformation errors affect the process skill and encoding stages; calculations performed using incorrect procedures yield incorrect final results. Therefore, it can be concluded that Subject S1 did not make an error in the process skill and encoding stages; instead, the error was a direct result of the transformation-stage error.

*Subject of Medium Mathematics Ability S2*

When completing question 1, Subject S2 correctly interpreted the problem, enabling him to complete the reading stage. Furthermore, Subject S2 was able to identify information and understand the question correctly, allowing him to complete the comprehension stage correctly. However, Subject S2 made errors in determining the solution procedure and in his calculations. This is shown in Figure 5.

| Merah                        | Biru                         |
|------------------------------|------------------------------|
| ① $P(A) = \frac{n(A)}{n(S)}$ | ② $P(A) = \frac{n(A)}{n(S)}$ |
| $= \frac{4}{9}$              | $= \frac{4}{9}$              |
| $= \frac{2}{3}$              | $= \frac{2}{3}$              |

**Figure 5. Subject S2's Answers**

On the answer sheet, Subject S2 noted that the number of elements in his sample space, or  $n(S)$ , was the initial number of balls, which was 9. However,  $n(S)$  should have changed to 8 after the first draw, considering that one ball had been drawn and not replaced. This is supported by the following excerpt from an interview.

*Q : What procedure or formula did you use to solve this problem?*

*S2 : The formula is  $P(a) = \frac{n(A)}{n(S)}$ .  $n(A)$  is the blue ball; there are five blue balls, and Diana took 1, making it 4. Then  $n(S)$  is the total number of balls in Diana's bag.*

In the interview, Subject S2 correctly used the probability formula; however, he misunderstood the concept of sample space. He assumed his sample space still consisted of the initial ball, with the number of elements of the sample space being 9, without taking into account the first draw that had been made. This indicates that Subject S2 did not fully understand the concept of sample space transformation in the case of drawing without replacement, leading to an error during the transformation stage.

In addition to transformation errors, Subject S2 also made a calculation error during the process skills stage. The process skills error occurred when Subject S2 incorrectly simplified the fraction  $4/9$  to  $2/3$  by dividing the numerator by 2 and the denominator by 3. In fact, when simplifying fractions, the numerator and denominator should be divided by the same number. The following excerpt from the interview confirms this.

*Q : So, how do you convert  $4/9$  to  $2/3$ ?*

*S2 : Divide the 4 by 2, and divide the 9 by 3.*

*Q : Are you sure?*

*S2 : Yes.*

Interviews with Subject S2 revealed that he believed he had simplified the fraction correctly, even though the procedure was incorrect. This indicates that Subject S2 did not yet understand the basic principles of fraction simplification.

At the encoding stage, Subject S2 was not considered to have made an error, even though his final answer was incorrect. This error in the final answer was caused by errors in the previous stages, namely, transformation and process skills. This indicates that Subject S2 lacked a grasp of the formula and the basic principles of fraction simplification, resulting in an incorrect final result.

## Student Errors in the Low Mathematics Ability Category

### Low Mathematics Ability Subject R1

When completing question 1, Subject R1 correctly interpreted the question, enabling him to complete the reading stage. Furthermore, Subject R1 was able to identify information and understand the question correctly, allowing him to complete the comprehension stage correctly. However, Subject S1 made errors in determining the solution procedure, making calculations, and failing to include a conclusion on his answer sheet. This is shown in Figure 6.

$$\begin{array}{l}
 P(A) = \frac{n(A)}{n(S)} \\
 n(A) \\
 P(A) = \frac{5}{4} \\
 P(S) = \frac{4}{2}
 \end{array}$$

Figure 6. Subject R1's Answers

On the answer sheet, Subject R1 was unable to explain the procedure or formula used to solve the problem. Subject R1 wrote down the answer without providing a clear explanation. To obtain a more complete explanation, the researcher conducted an interview, with the following results:

*Q : What procedure or formula do you think you used to solve the problem?*

*R1 : I don't know*

*Q : On your answer sheet, you wrote  $M = 5$ ,  $M = 4$ . Could you explain where this came from?*

*R1 : From the method on the board*

*Q : From the board? From notes?*

*R1 : Yes*

*Q : In your notes, are the problems the same?*

*R1 : Yes. Oh, they're different.*

*Q : They're different. So, you copied them, huh?*

*R1 : (Student nods)*

In the interaction, it was apparent that Subject R1 had difficulty explaining the procedures or formulas used and admitted copying answers from notes without paying attention to differences in the problem context. This indicates that Subject R1 was unable to transform the problem into the correct mathematical form given the situation, and thus could not determine the procedure for solving it. This caused Subject R1 to make a transformation error.

Furthermore, Subject R1 also made an error in the process skills stage. A process skills error occurred when Subject R1 made a calculation error. The subject explained how to simplify  $5/4$  by inverting it to get  $1/2$ . However,  $5/4$  should not be simplified to  $1/2$ . Furthermore, the procedure for simplifying fractions is to divide the numerator and denominator by the same number, not by inverting the fraction. The following excerpt from an interview supports this.

*Q : Here you say  $5/4 = 1/2$ . How do you calculate that?*

*R1 : From here, then reverse it (student points to  $5/4$ )*

*Q : Does  $5/4$  reverse to  $1/2$ ?*

*R1 : Yes*

*Q : Are you sure about your answer?*

*R1 : No*

*Q : Why?*

*R1 : Because the calculation was wrong*



*Q : Then what is the correct way?*

*R1 : I don't know*

In the interview, Subject R1 admitted that his calculations were incorrect. However, Subject R1 was unable to state the correct method for simplifying the fraction. Furthermore, the interview revealed that Subject R1 did not fully understand the rules for calculating fractions. This lack of understanding led Subject R1 to make errors at the process skill stage, particularly in performing mathematical calculations in accordance with applicable rules.

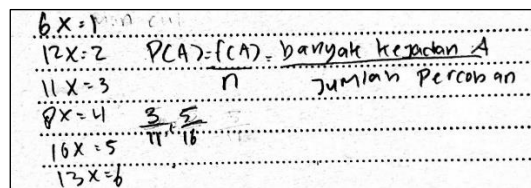
On the other hand, Subject R1 also made an error in the encoding stage. The encoding error occurred because Subject R1 was unable to determine a conclusion from his answer that matched the given problem. The following interview results reinforce this.

*Q : So, what is the conclusion of your answer?*

*R1 : I don't know.*

#### *Subject of Low Mathematics Ability R2*

When completing question 2, Subject R2 identified the information and understood the question correctly, thus completing the comprehension stage. However, Subject S1 made errors in interpreting the wording of the question, incorrectly applying the specified formula, and failed to include a conclusion on his answer sheet. These errors are shown in Figure 7.



The image shows a student's handwritten work on lined paper. On the left, there is a list of outcomes for a six-sided die: 6x=1, 12x=2, 11x=3, 8x=4, 16x=5, and 13x=6. On the right, the student has written the formula for probability:  $P(A) = \frac{f(A)}{n}$ . Below the formula, they have written 'banyak kejadian A' (number of occurrences of A) under  $f(A)$  and 'jumlah Peristiwa' (total number of events) under  $n$ . In the center, there is a calculation:  $\frac{3}{11} \cdot \frac{5}{16}$ .

**Figure 7. Subject R2's Answers**

On the answer sheet, Subject R2 demonstrated a reading error by incorrectly interpreting "prime numbers". This error was apparent when Subject R2 stated that the prime numbers were 3 and 5. However, a prime number is a positive integer with exactly two factors: 1 and the number itself. Therefore, the prime numbers on the dice are 2, 3, and 5. The following interview quote supports this statement.

*P : Okay, so what's the next step?*

*R2 : Um, the prime numbers are 3 and 5. I'll look for the first 3, and it's three over 11. If five is five over 10, it's five over 10.*

The interview results showed that Subject R2 misinterpreted the term "prime number" in the context of the problem. Furthermore, Subject R2 also demonstrated an error in the transformation stage. Transformation errors occur when Subject R2 incorrectly applies the specified formula. Subject R2 explained that the way to calculate the probability of a 3 is by dividing 3 (the number of dice 3) by 11 (the number of times a 3 appears), and the same applies for a 5. Subject R2 should have divided the number of times a prime number (2, 3, and 5) appeared, which is 33, by the number of times all the dice appeared, which is 60, to obtain  $33/60$ . The following excerpt from an interview supports this statement.

*P : Okay, explain where the numbers 3 and 11 come from.*

*R2 : 3 is a number three divided by 11, and 11 is the number of times a five appears.*

*P : Okay, is the method the same for five by 10?*

*R2 : Same here.*

The interview revealed that Subject R2 made an error in applying the formula based on the information in the problem. However, on the answer sheet, Subject R2 correctly determined the formula. This reflects a transformation error, where Subject R2 was unable to apply his knowledge to solve the problem correctly.

Transformation errors affect the process skill and encoding stages; calculations performed using incorrect procedures yield incorrect final answers. Therefore, it can be concluded that Subject R2 did not make an error at the process skill and encoding stages; instead, the error was a direct result of the mistake at the transformation stage.

This study found four errors across all mathematical ability categories: reading, transformation, process skills, and encoding. The errors that occurred in each mathematical ability category are presented in Table 4. Based on Table 4, all students in this study did not make errors at the comprehension stage. All students correctly identified known and requested information. This study also noted that, although students had varying levels of mathematical ability, errors persisted in the probability material. The errors found in students with high mathematical ability were at the process skill stage.

Furthermore, students with moderate mathematical ability found errors at the transformation and process skill stages. Meanwhile, students with low mathematical ability found errors at the reading, transformation, process skills, and encoding stages. This indicates that students with low mathematical ability tend to make more errors. These results are consistent with Siswandi (2021) findings that students with low mathematical ability tend to make more errors than students with high mathematical ability.

**Table 4. Mistakes in Each Category of Mathematical Ability**

| Number | Tahapan Newman       | High |    | Medium |    | Low |    |
|--------|----------------------|------|----|--------|----|-----|----|
|        |                      | T1   | T2 | S1     | S2 | R1  | R2 |
| 1      | Reading error        | -    | -  | -      | -  | -   | ✓  |
| 2      | Comprehension error  | -    | -  | -      | -  | -   | -  |
| 3      | Transformation error | -    | -  | ✓      | ✓  | ✓   | ✓  |
| 4      | Process skill error  | ✓    | -  | -      | ✓  | ✓   | -  |
| 5      | Encoding error       | -    | -  | -      | -  | ✓   | -  |

In students with high mathematical abilities, no errors were recorded in the reading, comprehension, transformation, and encoding stages. High-ability students could correctly interpret the problem, explain the known and requested information, determine the approach or method used to solve the problem, and determine the final result. This aligns with the findings of Iriani, Sridana, Triutami, & Azmi (2022), who found that high-ability students can complete the reading, comprehension, and transformation stages correctly. Furthermore, research by Ahmad & Nasution (2018) indicates that high-ability students can accurately define problems and write information in mathematical form. However, high-ability students first made errors in the calculation stage, which is included in the process skill stage. Inaccurate calculations caused these errors. In line with previous research, these findings indicate that calculation errors are the main factor behind the errors students often make at the process skills stage (Cahyaningtyas, Rahardi, & Irawati, 2021; Murtiyasa & Wulandari, 2020; Rian, Ellissi, & Resi, 2023).

Students with moderate mathematical abilities showed errors at the transformation and process skill stages. Moderately able students correctly interpreted the problem and explained important information and the issues to be solved, so they did not make errors at the reading, comprehension, and encoding stages. This finding aligns with research by Iriani, Sridana, Triutami, & Azmi (2022), which found that moderately able students had strong reading and problem-solving skills. Students with moderate mathematical abilities were recorded as making transformation errors due to a lack of understanding of the formulas used in the context of taking without replacement. This finding is supported by research conducted by Kristianti, Anjarwati, Irsyadi, & Nisa (2023), which found that students made transformation errors due to a lack of understanding of the correct

formulas. Furthermore, process skill errors were also observed in students with moderate mathematical abilities due to a lack of knowledge of the basic principles of correctly simplifying fractions. Hartana, Yenni, & Hartantri (2023) reinforced this finding by showing that a lack of understanding of fraction arithmetic operations contributes to errors at the process skill stage.

Students with low mathematical abilities made errors in the reading, transformation, and encoding stages, as well as in process skills. The findings of this study are consistent with the results of research by Astuti, Setyawati, & Ayuwanti (2024), which showed that students with low mathematical abilities experienced difficulties in several stages of working on mathematics problems, namely the stages of reading the problem, transforming the problem, applying process skills, and writing the final answer. Students made reading errors with low mathematical abilities, and secondly, due to misinterpreting the words in the problem. This research finding aligns with the findings of Timo, Nahak, & Mamoh (2022), who reported that reading errors are caused by students misinterpreting words and terms in the problem. On the other hand, students with low mathematical abilities first made an error in the transformation stage because they were unable to transform the problem into a mathematical form, given the situation, and thus could not determine the procedure for solving it. This finding aligns with research by Murtiyasa & Wulandari (2020) and Timo, Nahak, & Mamoh (2022), which found that these errors occurred because students were unable to formulate the correct procedure.

Furthermore, errors were also found in the transformation stage made by students with low mathematical abilities. Transformation errors occurred because the formula was not applied correctly. This finding aligns with the research of Febryana, Sudiana, & Pamungkas (2023), which found that transformation errors occur when students enter elements into a predetermined formula incorrectly, even though they already know the correct formula.

Meanwhile, process skill errors were found in students with low mathematical abilities, primarily due to a lack of understanding of the rules for calculating fractions. This finding aligns with the research of Hartana, Yenni, & Hartantri (2023), which found that students can make errors in the process skill stage if they do not understand the concepts of fraction arithmetic operations. Meanwhile, encoding errors occur when the answer cannot be determined from the given problem. This is relevant to the research by Sari, Ferdiani, & Yuwono (2018), which found that the factor causing errors in writing answers is students' inability to determine the final answer based on the intended conclusion of the problem.

The implications of this research finding can be applied to learning strategies such as scaffolding. Scaffolding can structure the problem-solving process (Amelia, Rofiki, Tortop, & Abah, 2020). Based on the finding that student errors tend to occur at certain stages, scaffolding can be adjusted to help students successfully pass each stage. Reading errors are caused by misinterpreting words in the problem, so it is necessary to increase vocabulary by providing a list of mathematical words or terms with their definitions (Buhaerah, Nasir, Busrah, & Aras, 2023). In the transformation stage, it is essential to strengthen the understanding of basic formulas and concepts and increase practice problems on probability (Agustin, Murniasih, & Fayeldi, 2023; Hadaming & Wahyudi, 2022). In the Process Skills stage, reinforcing the concept of fraction arithmetic operations can help students correct errors and teach the technique of double-checking answers to ensure accurate calculations (Upu, Taneo, & Daniel, 2022). Finally, at the encoding stage, students need to be taught to determine conclusions by linking answers to questions (Lombasari, Subarinah, Azmi, & Kurniati, 2022).

## CONCLUSION

This study found errors in all four stages of Newman's mathematics (reading, transformation, process skills, and encoding) across all levels of mathematical ability. High-ability students experienced errors only in the process skills stage, due to calculation inaccuracies. Medium-ability students experienced errors in the transformation stage due to a lack of understanding of concepts

and formulas, and in process skills due to an insufficient knowledge of the principles of fraction simplification. Meanwhile, low-ability students made errors at all stages: reading (misinterpreting the problem), transformation (inability to convert the problem into mathematical form and apply the formula), process skills (fraction simplification errors), and encoding (inability to conclude the problem). The researchers recommend that teachers increase mathematical vocabulary, strengthen students' understanding of concepts and formulas, provide focused practice in fraction operations, and train students to draw appropriate conclusions. Future research is recommended to examine the use of scaffolding appropriate to the types of errors at each Newman stage.

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