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The Verbal Problem-Solving Process in Students' Mathematical Connections

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ABSTRAK. Penelitian ini mengkaji proses pembentukan koneksi matematis dalam menyelesaikan soal verbal dan berbasis cerita. Berbagai jenis koneksi berhasil diidentifikasi oleh siswa saat menyelesaikan soal verbal. Jenis-jenis soal dikembangkan berdasarkan kerangka kerja NCTM, meliputi koneksi antar topik matematika, koneksi dengan disiplin ilmu lain, dan koneksi dengan konteks kehidupan sehari-hari. Data dikumpulkan melalui wawancara semi-terstruktur dengan dua subjek yang dipilih menggunakan teknik purposive sampling. Dalam prosesnya, siswa menunjukkan tujuh jenis koneksi: koneksi pemahaman, koneksi if-then, koneksi ekuivalensi, koneksi hierarkis, koneksi perbandingan umum, koneksi prosedural, dan koneksi justifikasi. Koneksi ini menggambarkan bagaimana ide-ide matematika ditransformasikan dan diekspresikan selama proses pemecahan masalah.

Kata Kunci: koneksi matematis: pemecahan masalah; soal verbal

ABSTRACT. This study examines the process of building mathematical connections when solving verbal and story-based mathematical problems. Students identified various types of connections while solving verbal problems. The question types were developed based on the NCTM framework, including connections between mathematical topics, connections with other disciplines, and connections to real-life contexts. Data were collected through semi-structured interviews with two subjects selected through purposive sampling. During the process, students demonstrated seven types of connections: understanding connections, if-then connections, equivalence connections, hierarchical connections, general formal comparison connections, procedural connections, and justification connections. These connections illustrate how mathematical ideas are transformed and expressed during problem-solving.

Keywords: mathematical connection; problem solving; verbal problems

INTRODUCTION

Problem-solving is a complex process that involves cognitive, emotional, and motor functions (Ridwan et al., 2017). During the problem-solving process, students interpret the problem, gather relevant data, decide on solutions, present those solutions, and evaluate their appropriateness (Aini et al., 2019; Hadinugrahaningsih et al., 2017). This suggests that problem-solving does not end with finding solutions; rather, understanding the problem is the foundational step for scientifically evaluating the solutions (Handajani et al., 2018; Wartono et al., 2018). According to Hong and Diamond, the scientific problem-solving process includes estimation, observation, information processing, and result generation. Using a structured problem-solving method allows students to practice advanced thinking and communication skills simultaneously.

Understanding the problem, determining the necessary data for a solution, and assessing the solution's validity are all critical stages in the scientific problem-solving (Saputra et al., 2019; Surya & Syahputra, 2017). Problem-solving requires combining these processes and applying them

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effectively to address the challenge at hand (Belecina & Jose M Ocampo, 2018; Purba et al., 2017). Polya emphasized that problem-solving is a process of discovering meaning until it can be fully understood (Butt et al., 2020; Daulay & Ruhaimah, 2019; Narita, 2019). It involves navigating obstacles, overcoming challenges, and using appropriate resources to achieve a goal. In this study, the problem-solving method is based on Polya's framework, which includes analyzing the problem, planning a solution, implementing the solution, and reviewing the results.

To effectively solve problems, connections between the stages of the problem-solving process are essential (Tohir et al., 2020). This highlights the integral role of mathematical connections in students' problem-solving experiences. Mathematical connections enable students to develop a conceptual understanding by linking interconnected mathematical concepts to solve problems. Students require these connection skills to relate mathematical ideas not only to other mathematical concepts but also to interdisciplinary topics and real-life situations.

Building mathematical connections involves linking ideas, concepts, or procedures within mathematics (Kidron & Dreyfus, 2010). When mathematical ideas are interconnected, students can identify the fundamental principles underlying the knowledge they possess (Pratiwi et al., 2020). A mathematical connection represents the relationships between mathematical concepts or between mathematics and other disciplines (Prayitno, 2018). These connections often manifest as equivalent representations of mathematical concepts.

From the perspective of constructivist theory, connections arise naturally as a form of conceptual understanding. These connections form a structured network, akin to a spider's web, where nodes represent pieces of information and the links between them signify connections (Chamberlin, 2005; Warner & Kaur, 2017). Thus, mathematical connections can be described as components within a schema or a network of related schemas in a mental framework. Schemas, in turn, are memory structures developed through individual experiences and interactions with the environment.

When students connect mathematical ideas, their understanding deepens and becomes more enduring. They begin to perceive mathematics as an interconnected and coherent discipline. Constructivist theory views mathematical connections as bridges that integrate prior knowledge with new insights, enhancing understanding of relationships between mathematical ideas, concepts, and representations. Based on this understanding, this research aims to identify the types of connections that students develop during the verbal problem-solving process.

One approach to assessing students' problem-solving abilities is by presenting them with complex problems (Daulay & Ruhaimah, 2019; Rahayuningsih et al., 2020). Verbal problems in mathematics, which require applying abstract concepts to real-world scenarios, are examples of such challenges. These problems are categorized as complex because they often require higher-order thinking. Solving problems with characteristics of complexity and openness has been shown to enhance mathematical reasoning and problem-solving skills (Arfiani et al., 2020; Habibi et al., 2020; Sunanto et al., 2020). Verbal problems are integral to mathematics education because they encourage students to link mathematical reasoning with real-life knowledge.

Solving story-based problems helps students develop the ability to apply mathematical knowledge effectively in daily life (Kovari & Rajesanyi-Molnar, 2020). These problems often involve tasks embedded in real-world contexts, requiring students to unpack the situation, extract mathematical principles, and solve the problem (Aidossov et al., 2021). Despite their importance, the use of mathematical connections in solving verbal problems is still not well understood by many students.

Several studies have investigated mathematical connections in problem-solving. For instance, Supratman et al. (2020)explored the relationship between justification and representation in high school students' problem-solving processes. Similarly, Cho & Kim (2020) examined how students' reasoning while solving poorly organized problems aligns with their epistemological understanding. However, there remains a gap in understanding how mathematical connections are utilized in solving verbal problems, especially those involving real-world contexts.

This study seeks to identify the connections that students establish during verbal problem-solving. Building these connections is crucial, as they reduce the cognitive load by linking relevant concepts and processes, thereby minimizing the reliance on memory alone (Purba et al., 2017). By understanding the various types of verbal connections, students can strengthen their problem-solving skills and expand their conceptual knowledge. This research aims to describe the relationships formed during verbal problem-solving and investigate how these connections contribute to students' mathematical understanding.

METHOD

This study employed a qualitative approach to explore the connections that emerge during verbal problem-solving. Semi-structured interviews were conducted to comprehensively characterize the connection processes that students demonstrate. The objective was to identify the types of connections that arise from observed phenomena (Bernard & Setiawan, 2020). The researchers developed a holistic understanding of the phenomenon by analyzing the words, in-depth explanations, and opinions of the respondents while conducting the investigation in a real-world setting. Students were asked verbal questions, allowing the researchers to capture the various types of linkages formed during the problem-solving process. Sufficient time was provided for the students to complete the tasks while responding to the questions.

The participants were seventh-grade students (Class VIIA) from a school in Parepare Regency, South Sulawesi, who had previously studied the topic of Two-Variable Linear Equation Systems (TVLES). A purposive sampling technique was applied, selecting individuals and settings most relevant to understanding the observed phenomena (Kidron & Dreyfus, 2010). Two students with high cognitive abilities were chosen as the research subjects based on their superior scores in an initial test and recommendations from their class teacher. The test questions were derived from previously studied material to ensure familiarity and relevance. The selected tasks on TVLES were designed to elicit a range of mathematical connections during the problem-solving process.

The verbal problems given to the participants were developed based on the three types of connections outlined by the National Council of Teachers of Mathematics (NCTM): (1) connections between mathematical topics, (2) relationships with other disciplines, and (3) connections to real-life contexts. The three verbal problems were as follows: (1) Problem 1**:** Teguh has some green marbles in Bag A and black marbles in Bag B. He transfers one-third of the green marbles into Bag B and half of the black marbles into Bag A. If Bags A and B contain 14 and 9 marbles, respectively, how many green marbles does Teguh have? (2) Problem 2: Ipul plans to give Tina a birthday gift, which will be placed in a rectangular box measuring 60 cm long, 20 cm wide, and 40 cm high. To make it visually appealing, Ipul wants to wrap the box with wrapping paper that measures 70 cm by 50 cm. How many sheets of wrapping paper will Ipul need to ensure the box is fully covered? (3) Problem 3: An athlete runs 50 laps around a rhombus-shaped park. If the diagonals of the rhombus measure 16 m and 30 m, respectively, what total distance does the athlete cover?

Each problem was designed to reveal specific indicators of mathematical connections. For example: (1) Problem 1: The subject demonstrated an *understanding connection* by identifying the known and required elements of the problem, then determining the appropriate concepts and procedures for solving it. Additionally, the subject applied an *if-then connection*, which involves logical reasoning and verification to derive conclusions based on given premises. Lastly, an *equivalent representation* emerged during the evaluation stage, where the subject represented the solution in different forms (e.g., verbal and symbolic). (2) Problem 2: The subject established a *hierarchical connection* by recognizing that one concept (the surface area of a rectangular box) is part of another concept (the paper needed to wrap the box). This connection illustrates how multiple mathematical concepts are interconnected within a hierarchy. (3) Problem 3: The subject applied the Pythagorean theorem to determine the length of the park's side, connecting this calculation to the total perimeter of the rhombus. This demonstrated the use of a procedural connection, where the subject linked multiple steps to arrive at a solution. Errors in calculating the diagonal length were identified and corrected during the evaluation stage, showing the subject's ability to revisit and refine their problem-solving process.

By analyzing the types of connections formed during problem-solving, this study aimed to gain deeper insights into how students link mathematical concepts across different contexts. The findings contribute to understanding the cognitive processes involved in verbal problem-solving and provide a basis for enhancing mathematical instruction.

RESULTS AND DISCUSSION

In the first problem, the subject employed an understanding connection. This connection was established based on the subject's ability to identify the known and asked elements of the problem to understand the concepts and procedures to be applied as a solution strategy. This ability was evident in how the subject linked all the known and asked elements in the problem. A solution strategy was then developed based on these connections. This process is illustrated in the following interview excerpt and Figure 1.

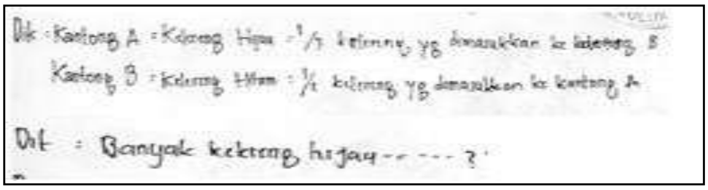

Figure 1. Subjects Identify the Known and Required Elements

Another type of connection identified was the "if-then" connection. This connection is closely related to generalization, though not exclusively, as it involves relationships built through verification or proof based on observation and logical reasoning. This connection emerged when the subject solved the problem using direct verbal analysis, drawing conclusions from previously known pieces of information. In mathematics, deductive reasoning or logic involves reaching conclusions based on known facts (premises). If the premises are true, the conclusion must also be true. This process is illustrated in Figure 2.

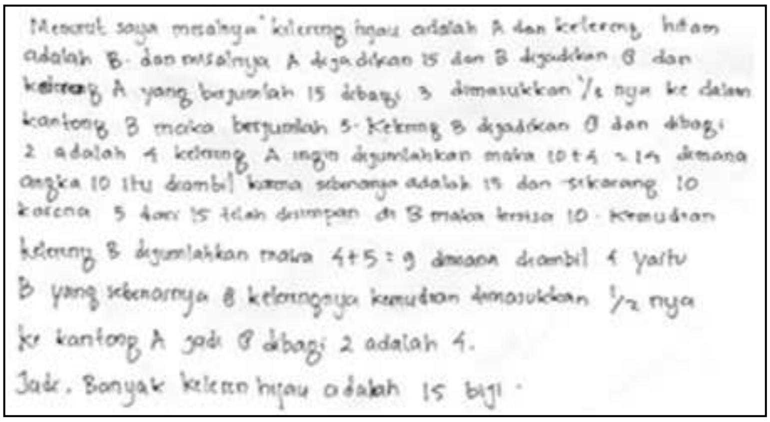

Figure 2. The Subject's Problem-Solving Process

Next, an equivalent representation connection was observed during the evaluation stage. Equivalent representation refers to a concept being represented in different ways but maintaining the same meaning, or one concept being represented using different symbols. This was evident in the subject's ability to verify the correctness of the answers obtained by employing alternative representations, specifically transitioning from verbal representations to symbolic representations. This process can be observed in the following interview excerpt.

In the second problem, the subject demonstrated a hierarchical connection. A hierarchical connection occurs when one concept is a component of another concept. This relationship was evident as the subject identified the appropriate concept to solve the problem. Specifically, the subject visualized the surface area of a rectangular prism representing the gift box and the square area representing the wrapping paper.

The comparison between the surface area of the prism and the area of the square paper was used as the basis for determining the amount of wrapping paper needed. This indicates that the subject established a hierarchical connection, where one concept forms a component of another. For instance, a rectangle is a part of a prism. This illustrates the subject's ability to make hierarchical connections.

In the third problem, the subject determined the side length of the garden by connecting it to the Pythagorean theorem. However, an error occurred in determining the diagonal length of the rhombus. Without recognizing the mistake, the subject continued the process by calculating the perimeter of the rhombus, which represented the athlete's running distance. The error was identified during the evaluation stage, after which the subject corrected the mistake. This process is illustrated in the following interview excerpt.

Furthermore, during the resolution of the third problem, the subject made a comparison connection through general forms. A comparison connection involves comparing two concepts with similar characteristics, allowing for comparisons based on their similarities or differences. This connection was evident in the subject's reasoning, where the garden was represented as a rhombus. This process is depicted in Figure 3.

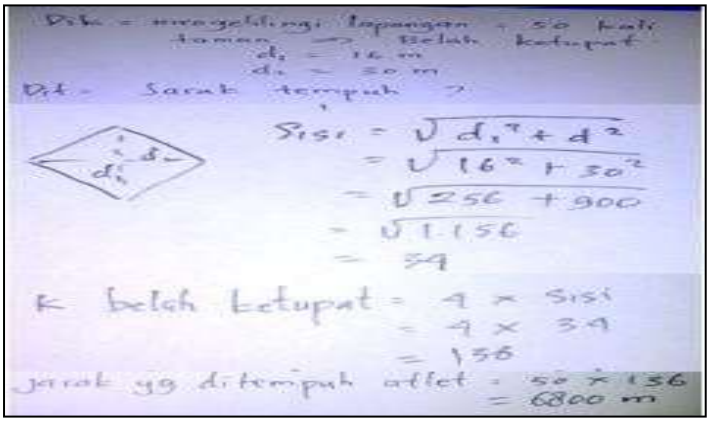

Figure 3. The Process of Addressing the Third Question

Figure 3 demonstrates that in addition to making a hierarchical connection, the subject also established a procedural connection. A procedural connection occurs when one concept is used as a process or method to connect to another. This was evident in how the subject used the Pythagorean theorem to calculate the perimeter of the rhombus. Typically, the connection between justification and representation is established during the problem-solving process for the three questions. The subject's ability to assess the correctness of the solutions using the employed concepts and procedures highlights this connection.

Mathematical connections naturally arise from constructivist theory as a form of conceptual understanding by creating structured networks. By connecting mathematical ideas, students develop a deeper and more enduring understanding of mathematics as an integrated whole. Verbal problems in mathematics, categorized as complex problems, are particularly effective in fostering such connections, as they require students to apply abstract mathematical concepts to real-world contexts.

The study results reveal the subject's ability to connect all known and asked elements in a problem, develop solution strategies, and implement multiple forms of reasoning and representation. These abilities indicate the presence of understanding, hierarchical, procedural, and comparison connections, demonstrating the potential of verbal problems to strengthen mathematical problem-solving skills.

CONCLUSION

The findings of this study reveal several types of connections made during the verbal problem-solving process. Understanding connections are established based on the subject's ability to identify the known and required components of a problem to determine the appropriate theories and strategies for resolution. If-then connections are observed when subjects employ direct verbal reasoning to draw conclusions from existing information, using logical reasoning and inference as a foundation.

Equivalent representation connections involve the ability to represent a concept in different ways while maintaining the same meaning. This was evident in the transition between verbal and symbolic representations. Hierarchical connections emerge when one concept is a component of another, such as when solving problems involving geometric relationships like rectangular prisms and squares.

Additionally, the study highlights comparative connections, where subjects compare and analyze similarities or differences between two concepts, often using general forms. Procedural connections are established when subjects use specific methods or techniques to link concepts during problem-solving. Finally, connections between justification and representation occur when subjects assess the accuracy of their answers using the concepts and methods applied.

These findings demonstrate that mathematical connections are integral to students' problem-solving processes, as they facilitate deeper understanding and improved reasoning skills. Encouraging the development of such connections can significantly enhance students' ability to solve complex mathematical problems and apply mathematical concepts to real-world scenarios.

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