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Fuzzy Single Depot mTSP Model using Robust Ranking Technique for Handling Deposit Carrying Bank

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Abstract

Multiple Traveling Salesman Problem (mTSP) of assignment-based consists of two types, namely the single-depot and multi-depot. This study aims to develop a single-depot mTSP assignment-based model with fuzzy travel cost form. The single-depot mTSP model above was formulated using an objective function with trapezoidal fuzzy-coefficient form. The fuzzy forms above were converted into crisp using the Robust Ranking Technique for getting an optimal solution. The developed model above was applied to handle deposit-carrying problem at Mandiri Bank with 20 branches in Pekanbaru, Riau Province, Indonesia. In this problem, the main objective is to minimize the total travel cost by bank's salesmen from initial depot to all destination branches. The result indicated that the developed fuzzy single-depot mTSP model is capable to determine the minimum total cost above into IDR 70,980.00 with m= 4 salesmen,the upper bound $L = 6$ *and the lower bound* $K = 4$*. This developed model could be considered and enhanced in handling deposing-carrying problem from another sectors.*

Keywords: *trapezoid fuzzy number, single depot mTSP, Robust Ranking Technique*

1. Introduction

The Traveling Salesman Problem (TSP) is a classic problem that tries to find the Hamilton Circuit that a salesman $(m = 1)$ goes through who wants to visit several places (cities) exactly once and return to the city of departure with the aim of minimizing distance, time, or travel costs. The generalization of TSP problems is multiple Traveling Salesman Problem (mTSP). The mTSP problem is a problem of distributing several salesmen $(m \geq 2)$ from a company to perform certain tasks to various places (cities). In addition, the problem of mTSP can also be seen in goods distribution applications. As well as it takes more than one freight vehicle to distribute goods to different cities. In this case, the vehicle corresponds with the salesman. In mTSP, vehicles numbering *m* will visit all existing nodes (cities) with minimum cost or minimum distance.

The problem of mTSP has many variations, one of which is the assignment-based single depot mTSP formulation discussed by Kara and Bektas. The purpose of the assignment-based mTSP single depot formulation in this study is to minimize travel costs incurred by salesmen so as to minimize company expenses, mTSP research is progressing rapidly in terms of determining solutions to mTSP problems with some specific algorithms [1]. In 2014, mTSP solutions were discussed with a new metaheuristic algorithm that is efficient and evolutionary [2]. Furthermore, in 2017 it also developed a new heuristic algorithm based on the shortest trajectory algorithm [3]. Then, in 2018 discussed the determination of mTSP solutions with the Permutation Invariant Pooling Network which combines new design elements in a set of graph networks [4]. In 2019, the solution to the mTSP single depot problem was discussed with a new variant of the Open Close Multiple Traveling Salesman Problem (OCMTSP) where all salesmen are categorized internally in this case called permanent and external in this case called outsourcing positioned at one depot [5].

One of the applications of the assignment-based single depot mTSP formulation is in the case of Deposit Carrying. In 2025, discussing the formulation of single depot mTSP in Deposit Carrying in the banking world [6]. In the banking world, there are many products and types of services provided by banks to customers. The purpose of providing bank services is to support and facilitate business activities to collect funds and distribute funds from the community and to the community. Customer funds collected at each bank branch office must be collected and managed at the central bank office. The step taken to collect customer funds at the central bank office is to arrange the scheduling of crews/workers who will be assigned from the central bank office to take deposits, which in this case is called Deposit Carrying, to various branch offices. Efficient scheduling planning of a crew to take deposits is an important step in the bank's company's operational planning process. The scheduling of crews visiting bank branch offices is expected to take the right travel routes with the aim of minimizing the distance traveled by employees so as to save travel costs.

The author's contribution to this study is to form a fuzzy-based assignment-based single depot mTSP formulation with the coefficients of the destination function, in this case the travel cost will be expressed as a trapezoidal fuzzy number referred to in [7]-[8]. This means, the cost of a trip that was originally fixed at a single point is expressed in four-point intervals. This is because in the phenomena of life, travel costs can be influenced by several factors such as road conditions, traffic flow, vehicle speed, fuel consumption conditions, vehicle tire conditions and so on. The travel cost expressed as a trapezoidal fuzzy number will be constructed to a *crisp* number with the Robust Ranking Technique approach [9] . Furthermore, the model formed was applied to the case study of *Deposit Carrying* carried out at Bank Mandiri Pekanbaru by designating the head office as *a single* depot (origin node) and several other Bank Mandiri Pekanbaru branch offices were declared as intermediate nodes that would be visited by salesmen.

2. Research Methods

This research is a literature study research that develops the theory that exists in the formulation Singles assignment-based mTSP depot discussed by Kara and Bektas into a new theory that produces a formulation Singles assignment-based mTSP depot with travel costs fuzzy [1]. Some literature reviews related such as assignment issues [2], Graph and Network Theory [11],[3], convex fuzzy sets and fuzzy arithmetic [4], trapezoidal fuzzy number [5], TSP and mTSP issues [6], and the Robust Ranking technique approach [7]. In the concept of constraints and limitations, in [8] explained that most of the existing settlement methods for TSP work with three main constraints, namely: degree constraints; subtour elimination constraint, and integrality constraints [15]. Furthermore, it is explained about the formulation of single depot mTSP based on assignment on [1], directional graphs, and some constraints related to the single depot formulation of mTSP [16] .

Definition 2.1. The single depot mTSP problem is defined on a directional complete graph with being a $G = (V, A)$; $V = \{1,2,\ldots,n\}$ set of n-nodes, and a set arcs is expressed by $A = \{(i,j)|i,j \in V, i \neq j\}$ and $C = c_{ij}$ is a distance matrix with respect to each $(i,j) \in A$. The cost matrix can be symmetrical, asymmetrical, or Euclidean. Suppose there is an m-salesmen who is in one depot then a tour will be searched in such a way that all salesmen start and end at the depot, each other node is visited exactly once in a single tour, the number of nodes visited by the salesman is determined at an interval, and the overall cost of visiting nodes is minimized.

The problem of the single depot mTSP model can be illustrated as shown in Figure 1 below.

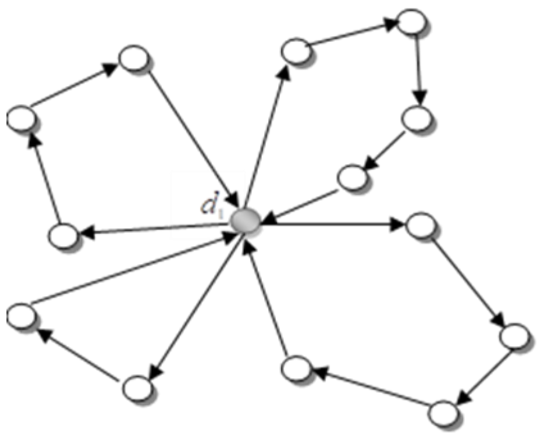

Figure 1. The single depot mTSP model.

Definition 2.2. x_{ij} a binary variable that is valued at 1 if $(i, j) \in A$ the solution is optimal and is 0 if the other. For any tour, u_i is the number of nodes visited on the visitor's trajectory from the original node to the node-i, L is the maximum number of nodes that the salesman can visit, up to $1 \leq u_i \leq L$, $i \geq 2$. Next, suppose there is K a minimum number of nodes that the salesman must visit, if $x_{i1} = 1$ then the conditions $K \leq u_i \leq L$ must be met.

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Proposition 2.1. Two boundary constraints expressed as

$$u_i + (L-2)x_{1i} - x_{i1} \leq L-1, \qquad i = 2,3,\ldots,n \tag{1}$$

$$u_i + x_{1i} + (2-K)x_{i1} \geq 2, \qquad i = 2,3,\ldots,n \tag{2}$$

is an inequality that applies to mTSP for $K \geq 2$.

If there is no minimum limit on the number of nodes that the salesman must visit, it is enough to replace the coefficient from x_{i1} within the constraint of Equation (1) with zero. Likewise, if the salesman is allowed to return to the depot after visiting only one node, the constraint of Equation (2) must be eliminated. For some cases, the lower limit of K may exist but there may be no restriction on the upper limit of L. In this case, assume that each vehicle (salesman) excludes one visit to the node $K(m-1)$. Then, the rest of $(n-1)-K(m-1)$ the nodes must be visited by the remaining number of vehicles. Thus, because $K \leq \lfloor (n-1)/m \rfloor$ substitution $(n-1)-K(m-1)$ replaces the value of L in the formulation.

Furthermore, the travel cost in this study is assumed to be a trapezoid fuzzy number which is defined as follows [13] :

Definition 2.2 Let α be a real number with $0 \leq \alpha \leq 1$ and let $\tilde{A}$ be a fuzzy set. The $\alpha - cut$ set of $\tilde{A}$, denoted by $\widetilde{A_\alpha}$, is defined as $\widetilde{A_\alpha} = \{x|\mu_{\tilde{A}}(x) \geq \alpha\}$.

Definition 2.3 A fuzzy numbers $\tilde{A}$ is called a trapezoidal fuzzy numbers and represented by (a_1, a_2, a_3, a_4) where a_1, a_2, a_3, a_4 are real numbers and the membership functions $\mu_{\tilde{A}}(x)$ are given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{untuk } x \leq a_1 \\ \frac{x-a_1}{a_2-a_1}, & \text{untuk } a_1 \leq x \leq a_2 \\ 1, & \text{untuk } a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3}, & \text{untuk } a_3 \leq x \leq a_4 \\ 0, & \text{untuk } x \geq a_4 \end{cases}. \tag{3}$$

To convert trapezoidal fuzzy numbers into crisp numbers, the Robust Ranking technique approach is used. Based on [14], suppose there are a fuzzy demand set, a fuzzy supply set, or a trapezoidal fuzzy cost set with $\tilde{A}= (a,b,c,d)$ is a trapezoidal fuzzy number with $a,b,c,$ and d is any real number then,

$$(L, U) = \{(b-a)\alpha + a + d - (d-c)\alpha\}, \tag{4}$$

If it is $\tilde{A}$ a trapezoidal fuzzy number, then Robust Ranking Technique can be defined as follows:

$$R(\tilde{A}) = \int_0^1 (0.5)(L, U)d\alpha, \tag{5}$$

with

$R(\tilde{A})$: Robust Ranking for trapezoidal fuzzy set $\tilde{A}$. The fuzzy set $\tilde{A}$ may represent a fuzzy demand set, a fuzzy supply set or a fuzzy cost set.

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$\int_0^1$: Integral with a limit of 0 to 1

(0.5) : The middle value of the interval [0. 1]

(L, U) : Calculation of the upper and lower limits of the fuzzy set $\tilde{A}$.

The steps in this study are explained as follows:

1. Let a single depot mTSP formulation based on assignment is given with the assumptions given by Kara and Bektas (2006) [15].
2. Declare the cost coefficient from node-i to node-j, denoted by c_{ij} in the objective function be a represented as a trapezoidal fuzzy number.
3. Reformulate assignment-based single-depot mTSP under the given assumptions, where $\tilde{c}_{ij}$ represents the trapezoidal fuzzy travel cost.
4. Constructing travel costs $\tilde{c}_{ij}$ into crisp numbers using the Robust Ranking technique.
5. Apply the new assignment-based single-depot mTSP formulation with trapezoidal fuzzy travel costs to the Deposit Cayying case study at Bank Mandiri, Pekanbaru.
   a. Data analysis
   The fuzzy travel costs $\tilde{c}_{ij}$ salesman passes between two independent bank offices in Pekanbaru that connect two nodes (two banks) are taken based on the consideration of the travel costs that occur when the salesman carries out his duties when taking deposits at each branch office. These data are assumed and expressed as trapezoid fuzzy numbers in Equation (1). Furthermore, the crisp value c_{ij} of this is determined using the Robust Ranking technique approach in Equation (4)-(5) and the obtained must meet the triangular inequalities with $c_{ij} \leq c_{ik} + c_{kj}, i \leq k \leq j;\ i, k, j \in V$.
   b. Furthermore, the map of the nodes of the Bank Mandiri Pekanbaru office was taken through Google Satellite or Google Maps.
   c. Given a set of nodes V (consisting of n-nodes) which in this case can be a single depot along with nodes that are intermediate nodes $(n-1)$ and a set of arcs A which contains arcs that connect two nodes, namely two different offices with each having a cost weight on each side. The nodes in the set V are first numbered from 1 until n, with $n \in N$.
   d. Assign the origin node or depot in this case the head office of the bank as the place that sends the salesman to visit the other nodes, in this case the branch offices (*intermediate nodes*).
   e. Assign a fixed number of m-salesmen to visit branch offices (intermediate nodes) starting from the origin node.
   f. Sets the lower limit K of a job or minimum node that the salesman must visit with the given formula $2 \leq K \leq \left\lfloor \frac{(n-1)}{m} \right\rfloor$.
   g. Setting the upper limit L of a job or the maximum node that the salesman can visit, namely $L \geq K$.

h. Clustering for the territory of intermediate nodes corresponds to the upper and lower boundaries of nodes that salesman can visit. This constitutes an additional operational constraint imposed during the assignment process.
i. Set u_i the number of nodes to visit on the traveler's path from the origin node to the node-i.
j. Making an assignment-based single depot mTSP formulation model knows the number of n-nodes, m-salesman, u_i the number of nodes visited on the traveler's path, with the coefficient c_{ij} representing the crisp value of a trapezoidal fuzzy number, then a decision variable x_{ij} with a value of 1 if the arc (i, j) solution and 0 if else. Furthermore, the optimal settlement of the case was determined using *the* LINGO 18 software.

3. Results and Discussion

The formulation of the integer program for the assignment-based mTSP single depot model with fuzzy travel costs is as follows:

Minimize $\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij} \qquad \tilde{c}_{ij} = (a, b, c, d), \qquad i \neq j, (i, j) \in A$ (6)

with constraints

$$\sum_{j=2}^{n} x_{ij} = m, \tag{7}$$

$$\sum_{j=2}^{n} x_{j1} = m, \tag{8}$$

$$\sum_{i=1}^{n} x_{ij} = m, \qquad j = 2, \ldots, n \tag{9}$$

$$\sum_{j=i}^{n} x_{ij} = m, \qquad i = 2, \ldots, n \tag{10}$$

$$u_i + (L-2)x_{1i} - x_{i1} \leq L-1, \qquad i = 2, \ldots, n \quad i = 2, \ldots, n \tag{11}$$

$$u_i + x_{1i} + (2-K)x_{i1} \geq 2, \qquad i = 2, \ldots, n \quad i = 2, \ldots, n \tag{12}$$

$$x_{1i} + x_{i1} \leq 1, \qquad i = 2, \ldots, n \quad i = 2, \ldots, n \tag{13}$$

$$u_i - u_j + Lx_{ij} + (L-2)x_{ji} \leq L-1, \qquad 2 \leq i \neq j \leq n \quad i = 2, \ldots, n \tag{14}$$

$$x_{ij} \in \{0, 1\}, \qquad \forall (i, j) \in A. \tag{15}$$

with

$\tilde{c}_{ij}$: the weight is a trapezoidal fuzzy number
m : the number of Salesmen
u_i : the number of nodes visited on that traveler's path from the oringin up to node-i
u_j : the number of nodes visited on that traveler's path from the oringin up to node-j
L : the upper (maximum) limit of the nodes to be visited
K : the lower limit (minimum) of nodes to be visited.

The assignment-based single depot mTSP formulation model in Equation (6)-(15) is applied in the case of Deposit Carrying at Bank Mandiri Pekanbaru which consists of 1 head office and 20 branch offices. In the banking world, there are many products and types of services provided by banks to customers. The purpose of providing bank services is to support and facilitate business activities to raise funds and distribute funds from and to the community. Customer funds collected at each bank branch office must be collected and managed at the bank's head office. The step taken to be able to collect customer funds at the central bank office is to arrange the scheduling of crews who will be assigned from the central bank office to take deposits, which in this case are called *deposits carrying* to various branch offices.

Efficient scheduling planning of a crew to take deposits is an important step in the bank's company's operational planning process. The scheduling of crews who visit bank branch offices is expected to take the right traveler's path so as to minimize the travel costs incurred by the bank company. Crew scheduling in this case consists of two stages, namely the stage of determining the number of crew and crew assignment. The determination of the number of workers to be assigned is fixed from the beginning and the assignment of the crew is based on the existence of a lower limit of visits in the trajectory taken. This lower limit arises because each worker is required to quit by at least one branch office to take deposits. The distribution of Bank Mandiri Pekanbaru offices is taken through Google Satellite and can be seen in Figure 2 below.

Figure 2. Distribution of Bank Mandiri branch locations in Pekanbaru.

In this case, the Bank Mandiri offices in Pekanbaru are defined as nodes $V = \{1,2,\ldots,21\}$; $n = 1,2,..,21$. We define the origin node or depot, which in this problem is the bank's head office, namely KC SB (Node 1), as the location that dispatches the salesmen (crew) to visit its branch offices as intermediate nodes, numbered Node 2, Node 3, and so on up to Node 21. Figure 3 presents the node numbering scheme.

Figure 3. Node labeling of Bank Mandiri offices in Pekanbaru.

Next, the number of salesmen is set to $m = 4$. The lower bound K denotes the minimum number of nodes that must be visited by each salesman, defined by $2 \leq K \leq \left\lfloor \frac{n-1}{m} \right\rfloor$, which gives $2 \leq K \leq 5$. Selecting $K = 4$ means that each salesman must visit at least 4 nodes. Then, the upper bound L which represents the maximum number of nodes that may be visited by each salesman, is defined such that $L \geq K$. In this case, $L = 6$ is specified. The next step is to cluster the intermediate nodes according to the specified lower and upper bounds on the number of nodes that each salesman is allowed to visit.

To simplify the solution of this problem, the manager of the Bank Mandiri Pekanbaru head office introduces an additional operational constraint by partitioning the service areas to be visited by the workers into four clusters. The first cluster contains four intermediate nodes, namely nodes 2, 3, 4, and 5. The second cluster contains five intermediate nodes, namely nodes 6, 7, 8, 9, and 10. The third cluster contains five intermediate nodes, namely nodes 11, 12, 13, 14, and 15. The fourth cluster contains seven intermediate nodes, namely nodes 1, 16, 17, 18, 19, 20, and 21. The resulting clustered regions are illustrated in Figure 4.

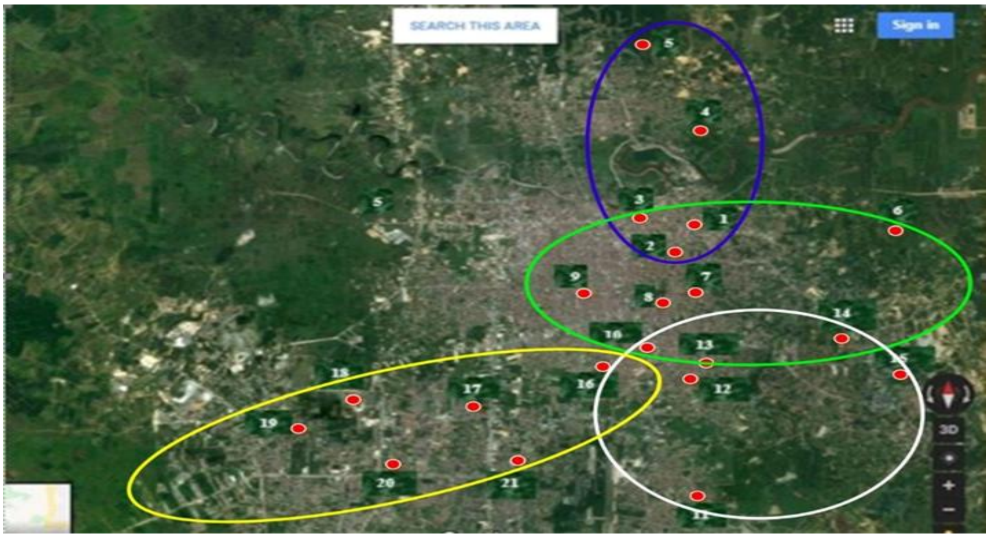

Figure 4. Cluster regions.

For each cluster, the corresponding subgraph is modeled as a complete directed graph that includes all intermediate nodes in the cluster and the depot, as illustrated in Figure 5.

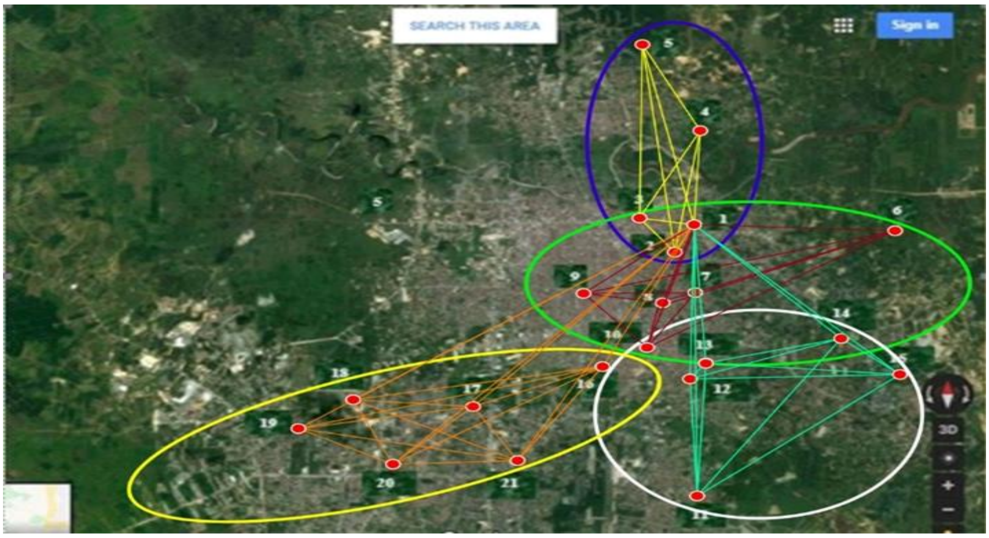

Figure 5. Complete directed graph

In this study, it is assumed that each salesman of Bank Mandiri Pekanbaru uses a Xenia 1000cc vehicle with a fuel consumption rate of 15 km/liter. Accordingly, the transportation cost associated with visiting all branch offices, which corresponds to the weights of the intermediate nodes in the mTSP model, is represented by a trapezoidal fuzzy number as defined in Equation (3).

In Cluster-1, there are 25 cost data $\tilde{c}_{ij}$with $i, j = 1,2,3,4,5$.
In Cluster-2, there are 36 cost data $\tilde{c}_{ij}$with $i, j = 1,6,7,8,9,10$.

In Cluster-3, there are 36 cost data $\tilde{c}_{ij}$ with $i, j = 1,11,12,13,14,15$.
In Cluster-4, there are 49 cost data $\tilde{c}_{ij}$ with $i, j = 1,16,17,18,19,20,21$.

Next, these travel cost data $\tilde{c}_{ij}$ are converted into crisp values using the Robust Ranking technique. As an example, in Cluster-1, it is known that

$$\tilde{c}_{12} = (a, b, c, d) = (655,658,662,665).$$

To determine the values (L, U) for node (1,2) using Equation (4), the following results are obtained.

$$\begin{aligned}(L, U) &= \{(b-a)\alpha + a + d - (d-c)\alpha\} \\ &= \{(658-655)\alpha + 655 + 665 - (665-662)\alpha\} \\ &= 3\alpha + 1320 - 3\alpha \\ &= 1320\end{aligned}$$

Furthermore, by substituting the value (L, U) into Equation (5) is obtained

$$\begin{aligned}R(\widetilde{c_{12}}) &= \int_0^1 (0.5)(L, U)d\alpha \\ &= \int_0^1 (0.5)(1320)d\alpha \\ &= \int_0^1 (660)d\alpha \\ &= 660(1) \\ &= 660\end{aligned}$$

Thus, the Robust Ranking score for node (1,2) is obtained as 660. Using the same procedure, this calculation is performed to obtain the crisp values of all fuzzy cost data for Cluster 1, Cluster 2, Cluster 3, and Cluster 4.

In the next stage, the value of the variable u_i, which represents the number of nodes visited on that traveler's path from the oringin up to node-i, is determined as follows :

$$u_{02} = 4, u_{03} = 3, u_{04} = 2, u_{05} = 1, u_{06} = 5, u_{07} = 4, u_{08} = 3, u_{09} = 2, u_{10} = 1,$$

$$u_{11} = 1, u_{12} = 4, u_{13} = 2, u_{14} = 3, u_{15} = 5, u_{16} = 6, u_{17} = 2, u_{18} = 4, u_{19} = 5,$$

$$u_{20} = 3, u_{21} = 1.$$

Thus, the assignment based single depot mTSP model with fuzzy travel costs, based on Equations (6)-(15), is formulated as follows:

Define: $x_{ij} = \begin{cases} 1, & arc(i,j) \text{ optimal solution} \\ 0 & \text{if others} \end{cases}$

Minimize

$$\sum_{i=1}^{21}\sum_{j=1}^{21} c_{ij}x_{ij} = 660x_{01,02} + 780x_{01,03} + 3000x_{01,04} + 5040x_{01,05} + 1440x_{01,06} + 900x_{01,07} + 1080x_{01,08}$$
$$+2400_{01,09} + 2220x_{01,10} + 4860x_{01,11} + 4080x_{01,12} + 2580x_{01,13} + 3840x_{01,14} + 4380x_{01,15} + 3060x_{01,16}$$
$$+3600x_{01,17} + 7440x_{01,18} + 7980x_{01,19} + 8580x_{01,20} + 6600x_{01,21} + 600x_{02,01} + 840x_{02,03} + 2880_{02,04}$$
$$+5040x_{02,05} + 1500x_{03,01} + 900x_{03,02} + 2340x_{03,04} + 4320x_{03,05} + 3660x_{04,01} + 3120x_{04,02} + 2460x_{04,03}$$
$$+1620x_{04,05} + 5580x_{05,01} + 5100x_{05,02} + 4380x_{05,03} + 1620x_{05,04} + 1440x_{06,01} + 1740x_{06,07} + 1800x_{06,08}$$
$$+3180x_{06,09} + 2880x_{06,10} + 900x_{07,01} + 1740x_{07,06} + 240x_{07,08} + 1680x_{07,09} + 1320x_{07,10} + 1080x_{08,01}$$

$+2100x_{08,06} + 240x_{08,07} + 1500x_{08,09} + 1140_{08,10} + 2460x_{09,01} + 3600x_{09,06} + 1800x_{09,07} + 1620x_{09,08}$
$+2100x_{09,10} + 2520x_{10,01} + 2880x_{10,06} + 1680x_{10,07} + 1500x_{10,08} + 1920x_{10,09} + 5520x_{11,01} + 2880x_{11,12}$
$+2400x_{11,13} + 4620x_{11,14} + 4440x_{11,15} + 2640x_{12,01} + 2700x_{12,11} + 10,8x_{12,13} + 2220x_{12,14} + 2220x_{12,15}$
$+2880x_{13,01} + 2400x_{13,11} + 10,8x_{13,12} + 3180x_{13,14} + 2220x_{13,15} + 3840x_{14,01} + 3900x_{14,11} + 3720x_{14,12}$
$+3180x_{14,13} + 540x_{14,15} + 4380x_{15,01} + 4440x_{15,11} + 3720x_{15,12} + 2220x_{15,13} + 540x_{15,14} + 3060x_{16,01}$
$+1080x_{16,17} + 5820x_{16,18} + 7140x_{16,19} + 7020x_{16,20} + 4260x_{16,21} + 3060x_{17,01} + 660x_{17,16} + 4740x_{17,18}$
$+6120x_{17,19} + 4920x_{17,20} + 5100x_{17,21} + 7440x_{18,01} + 5760x_{18,16} + 5220x_{18,17} + 1380x_{18,19} + 2760x_{18,20}$
$+4500x_{18,21} + 7980x_{19,01} + 6900x_{19,16} + 6420x_{19,17} + 1440x_{19,18} + 2580x_{19,20} + 4320x_{19,21} + 9600x_{20,01}$
$+7020x_{20,16} + 7080x_{20,17} + 2460x_{20,18} + 2340x_{20,19} + 1800x_{20,21} + 7500x_{21,01} + 5040x_{21,16} + 5100x_{21,17}$
$+3840x_{21,18} + 3900x_{21,19} + 1800x_{21,20}$

with constraints

$\sum_{j=2}^{21} x_{1j} = 4,$

$,\sum_{j=2}^{21} x_{j1} = 4$

$\sum_{i=1}^{21} x_{ij} = 1, \qquad j = 2,3, \ldots ,21.$

$\sum_{j=1}^{21} x_{ij} = 1, \qquad i = 2,3, \ldots ,21.$

$u_i + (L-2)x_{1i} - x_{i1} \leq L-1, \qquad i = 2,\ldots,21$

$u_i + x_{1i} + (2-K)x_{i1} \geq 2, \qquad K = 4, i = 2,\ldots,21$

$x_{1i} + x_{i1} \leq 1, \qquad i = 2,3, \ldots ,21.$

$u_i - u_j + Lx_{ij} + (L-2)x_{ji} \leq L-1, \qquad L = 6, 2 \leq i \neq j \leq 21$

$x_{ij} \in \{0,1\}, \forall (i,j) \in A.$

By using LINGO 18 software, the minimum total cost above into IDR 70,980.00 and the solutions to the assignment-based single-depot mTSP model are as follows:

$$x_{01,05} = x_{01,10} = x_{01,11} = x_{01,21} = x_{02,01} = x_{03,02} = x_{04,03} = x_{05,04} = x_{06,01} = x_{07,06} = x_{08,07} = x_{09,08} = x_{10,09} = x_{11,13} = x_{12,15} = x_{13,14} = x_{14,12} = x_{15,01} = x_{16,01} = = x_{17,20} = x_{18,19} = x_{19,16} = x_{20,18} = x_{21,17} = 1.$$

and

$$x_{01,02} = x_{01,03} = x_{01,04} = x_{01,06} = x_{01,07} = x_{01,08} = x_{01,09} = x_{01,12} = x_{01,13} = x_{01,14} = x_{01,15} = x_{01,16} = x_{01,17} = x_{01,18} = x_{01,19} = x_{01,20} = x_{02,03} = x_{02,04} = x_{02,05} = x_{03,01} = x_{03,04} = x_{03,05} = x_{04,01} = x_{04,02} = x_{04,05} = x_{05,01} = x_{05,02} = x_{05,03} = x_{06,07} = x_{06,08} = x_{06,09} = x_{06,10} = x_{07,01} = x_{07,08} = x_{07,09} = x_{07,10} = x_{08,01} = x_{08,06} = x_{08,09} = x_{08,10} = x_{09,01} = x_{09,06} = x_{09,07} = x_{09,10} = x_{10,01} = x_{10,06} = x_{10,07} = x_{10,08} = x_{11,01} = x_{11,12} = x_{11,14} = x_{11,15} = x_{12,01} = x_{12,11} = x_{12,13} = x_{12,14} = x_{13,01} = x_{13,11} = x_{13,12} = x_{13,15} = x_{14,01} = x_{14,11} = x_{14,13} = x_{14,15} = x_{15,11} = x_{15,12} = x_{15,13} = x_{15,14} = x_{16,17} = x_{16,18} = x_{16,19} = x_{16,20} = x_{16,21} = x_{17,01} = x_{17,16} = x_{17,18} = x_{17,19} = x_{17,21} = x_{18,01} = x_{18,16} = x_{18,17} = x_{18,20} = x_{18,21} = x_{19,01} = x_{19,17} = x_{19,18} = x_{19,20} = x_{19,21} = x_{20,01} = x_{20,16} = x_{20,17} = x_{20,19} = x_{20,21} = x_{21,01} = x_{21,16} = x_{21,18} = x_{21,19} = x_{21,20} = 0.$$

Based on these results, the optimal routes traveled by each salesman are obtained and presented in Table 1.

Tabel 1. Optimal Routes for Each Salesman

Cluster	Salesman	Route
1	A	1-5-4-3-2-1
2	B	1-10-9-8-7-6-1
3	C	1-11-13-14-12-15-1
4	D	1-21-17-20-18-19-16- 1

A solution graph of the assignment-based single depot mTSP model with fuzzy travel costs can be shown in the following Figure 6.

Figure 6. Solution Graph of the mTSP model

4. Conclusion

In this study, an integer formulation of the assignment-based single-depot mTSP model with fuzzy travel costs was developed and applied to a deposit carrying application at the Bank Mandiri Pekanbaru head office. The results indicate that the assignment-based single-depot mTSP model with fuzzy travel costs is capable of providing optimal solutions for distributing funds to 20 branch offices of Bank Mandiri in the Pekanbaru area. With the number of salesmen set to $m = 4$, the upper bound $L = 6$ and the lower bound $K = 4$, the total travel cost required to complete all routes amounts to only IDR 70,980.00. These findings demonstrate that the proposed model is effective in optimizing operational costs in a fund distribution scenario within the banking sector.

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